Supplementary material

In the supplementary material we present two robustness check. In the first robustness check, rent-seeking becomes less effective as agents spend more time rent-seeking, i.e. the activity has decreasing returns to scale. As we shall see, effectively, this reduces the return on rent-seeking and, in aggregate, there will be less rent-seeking. Since the tradeoff between income and inequality is unaltered, the effect is that aggregate income increases but is less evenly divided. The second robustness check, where monopolization occurs with probability less than one (or might even depend on the total amount of rent-seeking) stresses the same point.

The main changes to the model are as follows. The share function is modified to:

$$s(\eta) = \frac{\eta e(\eta)^{\varepsilon}}{R},$$

where $\varepsilon \in (0, 1]$. Note that $\varepsilon = 1$ is the case with constant returns to scale analyzed in the main text. The definition of R changes accordingly.

The other modification is that monopolization only happens with probability κ. With probability $1-\kappa$, both sectors are competitive and the price in sector 1 is also equal to the wage rate: $P_1^c = 1$. The true price index changes accordingly and is denoted by P_V^c.

The expected utility of agent η is denoted by:

$$\mathbb{E}[V(\eta)] = \kappa \frac{1 - e(\eta) + s(\eta)\Pi_1^m}{P_V^m} + (1 - \kappa)\frac{1 - e(\eta)}{P_V^c}, \tag{S.1}$$

where the true price index depends on whether monopolization occurs and, in case it does not, the agent only has labor income.

Robustness check 1: Decreasing returns to scale

This is the case where $\varepsilon < 1$, but we still have $\kappa = 1$. The price index is still irrelevant and an agent of type η choose $e \in [0, 1]$ to maximize:

$$1 - e + \frac{\eta e^{\varepsilon}}{R} \cdot \omega L \tag{S.2}$$

(cf. eq. 20). It follows that the interior solution (if it exists) is given by

$$e(\eta) = \left(\frac{\varepsilon \omega \eta L}{R}\right)^{1/(1-\varepsilon)} \tag{S.3}$$

Note that $e(\eta) > 0$ since the return on the first unit of rent-seeking is infinitely high for all agents. However at the top end of the distribution, agents may still choose to specialize fully in rent-seeking: with slight abuse of notation, suppose that $e(\eta) = 1$ if and only if $\eta > \hat{\eta}$.

First, suppose that $\hat{\eta} = \eta_1$ and there is an interior solution for every agent. Recall the definitions of R and L:

$$R = \int_{\eta_0}^{\eta_1} \eta e(\eta)^{\varepsilon} dF(\eta) \tag{S.4}$$

$$L = 1 - \int_{\eta_0}^{\eta_1} e(\eta) dF(\eta) \tag{S.5}$$

Plugging the expression for $e(\eta)$ into (S.4) and (S.5) we obtain:

$$R = \left(\frac{\varepsilon \omega L}{R}\right)^{\varepsilon/(1-\varepsilon)} \int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) \tag{S.6}$$

$$L = 1 - \left(\frac{\varepsilon \omega L}{R}\right)^{1/(1-\varepsilon)} \int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) \tag{S.7}$$

Note that (S.6) and (S.7) can be solved to get two expressions for the integral.

$$\int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{R}{\left(\frac{\varepsilon \omega L}{R}\right)^{\varepsilon/(1-\varepsilon)}} \tag{S.8}$$

$$\int_{\eta_0}^{\eta_1} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{1 - L}{\left(\frac{\varepsilon \omega L}{R}\right)^{1/(1-\varepsilon)}} \tag{S.9}$$

Obviously the right-hand side of (S.8) should be equal to the right-hand side of (S.9). If we equate both right-hand sides, then we see that R cancels out and we can solve it to obtain an expression for L. This leads to the following result:

Proposition S.1. *If $e(\eta) \in (0,1)$ for all η, then the size of the labor force is independent of the distribution of rent-seeking aptitude and given by:*

$$L = \frac{1}{1+\varepsilon\omega} \tag{S.10}$$

Since L is decreasing in ε, we see that the closer we get to constant returns, the smaller the labor force. The question is whether this conclusion also holds when $\hat{\eta} < \eta_1$.

Figure 6 shows a numerical example where η is uniformly distributed on the unit interval. Panel (a) depicts the rent-seeking effort as function of η for three different values of ε. It is clear that the effect of increasing ε on rent-seeking effort differs between types. At the top end of the distribution, it increases. At the bottom end, it decreases. For agents in the middle of the distribution, the effect is even non-monotonic. Note that, as ε increases, each incremental unit rent-seeking yields higher returns. So, *ceteris paribus*, rent-seeking is more profitable. However, on the whole, this reduces the productive labor force L (and monopoly profit) and increases the total amount of rent-seeking R. Both reduce the returns on rent-seeking, but the effect is larger for agents with low rent-seeking aptitude. In equilibrium, 'high η'-agents start to specialize in rent-seeking, while 'low η'-agents only spend a small fraction of their time on rent-seeking.

The net effect on L, for the case of the uniform distribution curve, is the area above the curve but below the $e = 1$-line in panel (a). It is clear that L decreases in ε: this is confirmed in panel (b), which shows L as function of ε. Note that the dashed line is the graph of equation (S.10): the reduction of the labor force is less severe than Proposition S.1 predicts.

The following Proposition confirms that our example generalizes to other distributions:

Proposition S.2. *Suppose $\hat{\eta} < \eta_1$. As ε increases and the rent-seeking technology gets closer to constant returns to scale, more agents become full time rent-seekers (i.e. $\hat{\eta}$ is decreasing in ε). However, for agents with low rent-seeking aptitude, this means increased competition for*

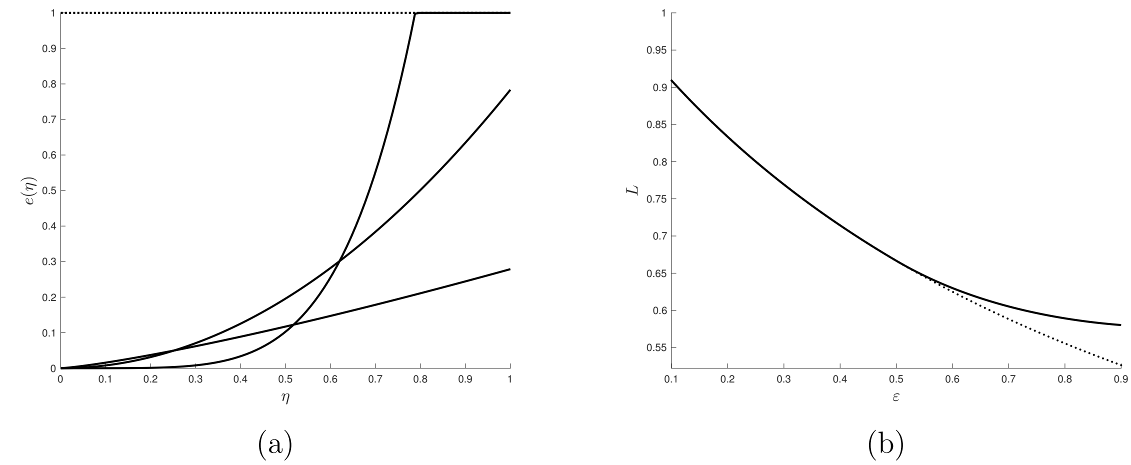

Figure 6: *Panel (a) shows time allocated to rent-seeking as function of type for different values of ε and panel (b) aggregate labor as function of ε (solid line), the dashed line shows aggregate labor for the case $\hat{\eta} = \eta_1$ when $L = 1/(1+\varepsilon\omega)$. Note that η is uniformly distributed on $[0, 1]$ and $\omega = \frac{1}{2}\sqrt{2}$.*

monopoly profit and their response is to work more. Consequently, the comparative statics of ε on L are ambiguous.

Proof. First of all, the expressions for R and L are different since some agents only engage in rentseeking:

$$R = \left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) + \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta), \tag{S.11}$$

$$L = F(\hat{\eta}) - \left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)} \int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta). \tag{S.12}$$

Note that, again, these two expression can be solved for the integral.

$$\int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{R - \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta)}{\left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)}}, \tag{S.13}$$

$$\int_{\eta_0}^{\hat{\eta}} \eta^{1/(1-\varepsilon)} dF(\eta) = \frac{F(\hat{\eta}) - L}{\left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)}}. \tag{S.14}$$

Therefore:

$$\frac{R-\int_{\hat{\eta}}^{\eta_1}\eta dF(\eta)}{\left(\frac{\varepsilon\omega L}{R}\right)^{\varepsilon/(1-\varepsilon)}}=\frac{F(\hat{\eta})-L}{\left(\frac{\varepsilon\omega L}{R}\right)^{1/(1-\varepsilon)}} \implies R=\frac{\varepsilon\omega L\int_{\hat{\eta}}^{\eta_1}\eta dF(\eta)}{(1+\varepsilon\omega)L-F(\hat{\eta})} \tag{S.15}$$

Next, observe that $e(\hat{\eta})=1$ implies that $\hat{\eta}=R/(\varepsilon\omega L)$. Taking the expression for R derived in (S.15), we obtain the following relationship between L and $\hat{\eta}$:

$$\hat{\eta}=\frac{\int_{\hat{\eta}}^{\eta_1}\eta dF(\eta)}{(1+\varepsilon\omega)L-F(\hat{\eta})}. \tag{S.16}$$

We can also use $\hat{\eta}=R/(\varepsilon\omega L)$ to eliminate R from (S.12):

$$L=F(\hat{\eta})-\hat{\eta}^{-1/(1-\varepsilon)}\int_{\eta_0}^{\hat{\eta}}\eta^{1/(1-\varepsilon)}dF(\eta). \tag{S.17}$$

Rewriting (S.16) and (S.17) slightly, we obtain the following implicit expression of L and $\hat{\eta}$ as function of ε:

$$(1+\varepsilon\omega)L\hat{\eta}-\hat{\eta}F(\hat{\eta})-\int_{\hat{\eta}}^{\eta_1}\eta dF(\eta)=0, \tag{S.18}$$

$$L-F(\hat{\eta})+\hat{\eta}^{-1/(1-\varepsilon)}\int_{\eta_0}^{\hat{\eta}}\eta^{1/(1-\varepsilon)}dF(\eta)=0. \tag{S.19}$$

Applying the implicit function theorem, we obtain

$$\begin{bmatrix}(1+\varepsilon\omega)\hat{\eta} & (1+\varepsilon\omega)L-F(\eta)\\ 1 & -\frac{1}{1-\varepsilon}\hat{\eta}^{-(2-\varepsilon)/(1-\varepsilon)}\int_{\eta_0}^{\hat{\eta}}\eta^{1/(1-\varepsilon)}dF(\eta)\end{bmatrix}\begin{pmatrix}dL/d\varepsilon\\ d\hat{\eta}/d\varepsilon\end{pmatrix}= \begin{pmatrix}-\omega L\hat{\eta}\\ -\frac{1}{(1-\varepsilon)^2}\int_{\eta_0}^{\hat{\eta}}\left(\frac{\eta}{\hat{\eta}}\right)^{1/(1-\varepsilon)}\log\left(\frac{\eta}{\hat{\eta}}\right)dF(\eta)\end{pmatrix}, \tag{S.20}$$

with signs:

$$\begin{bmatrix}(+) & (+)\\ (+) & (-)\end{bmatrix}\begin{pmatrix}dL/d\varepsilon\\ d\hat{\eta}/d\varepsilon\end{pmatrix}=\begin{pmatrix}(-)\\ (+)\end{pmatrix}. \tag{S.21}$$

From Cramer's Rule, it follows that $\hat{\eta}$ is decreasing in ε. The sign of $dL/d\varepsilon$ is ambiguous. ■

Robustness check 2: Probability of monopolization

We return to the case where $\varepsilon = 1$, but now we have $\kappa < 1$. As before, there is an indifferent agent $\hat{\eta}$ whose return to rent-seeking is the same as the wage rate: only those agents whose rent-seeking aptitude exceeds $\hat{\eta}$ specialize in rent-seeking. The agent with type $\hat{\eta}$ is indifferent between working and rent-seeking:

$$\kappa \frac{1}{P_V^m} + (1-\kappa)\frac{1}{P_V^c} = \kappa \frac{\hat{\eta}\omega L}{P_V^m R}, \tag{S.22}$$

where the left-hand side is obtained by evaluating (S.1) at $e = 0$ and the right-hand side by evaluating (S.1) at $e = 1$. Observe that there is a price effect where the higher prices in case of monopolization decrease indirect utility. Rearranging (S.22), we obtain:

$$1 + \frac{1-\kappa}{\kappa} \cdot \frac{P_V^m}{P_V^c} = \frac{\hat{\eta}\omega L}{R}. \tag{S.23}$$

The right-hand side is now identical to the analysis in Section 3, but the left-hand side shows a premium for workers equal to the odds against monopolization times the benefit of avoiding the monopoly price increase. This is not that surprising: when monopolization does not happen, workers unlike rent-seekers have a source of income and, as an additional bonus, prices are lower. The overall effect is less rent-seeking.[15]

This result continues to hold if the likelihood of monopolization depends on the aggregate amount of rent-seeking, i.e. $\kappa = \kappa(R)$. In particular, suppose that more rent-seeking increases the probability of monopolization: $R' > R$ implies $\kappa(R') > \kappa(R)$. We can think of it as a politician being persuaded by the shear amount of rent-seeking to grant monopoly rights in sector 1. For the individual agents, who take R as given, the incentives do not change drastically: in (S.23), κ simply becomes a function of R. However, at the macrolevel, there is a multiplier where rent-seeking begets more rent-seeking. This is a second-order effect that is dominated by the overall effect that monopolization no longer happens with certainty. Formally:

Proposition S.3. *Let κ be a continuous function of R with the properties that $\kappa(0) = \kappa_0 \in (0, 1)$, $\kappa(\mathbb{E}\eta) = 1$, and for all $R \in (0, \mathbb{E}\eta)$ we have $\kappa(R) \in [\kappa_0, 1)$. Then, in every equilibrium, rent-*

seeking is reduced compared to the benchmark where $\kappa \equiv 1$.

Proof. Suppose that in the benchmark, where monopolization always occurs, the indifferent agent is located at $\hat{\eta}_B$. Recall from (22) that

$$\int_{\hat{\eta}_B}^{\eta_1} \eta dF(\eta) - \omega \hat{\eta}_B F(\hat{\eta}_B) = 0. \tag{22''}$$

If κ is function of R, then this condition (cf. eq. S.23) changes to:

$$\left(1 + \frac{1-\kappa(\hat{\eta})}{\kappa(\hat{\eta})} \cdot \frac{P_V^m}{P_V^c}\right) \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega \hat{\eta} F(\hat{\eta}) = 0, \tag{S.24}$$

where κ is a function of $\hat{\eta}$ through R. Note that our assumptions on κ imply that $\kappa(\eta_0) = 1$ and $\kappa(\eta) \in (0,1) \forall \eta \in (\eta_0, \eta_1]$. Introduce

$$h(\hat{\eta}) = \left(1 + \frac{1-\kappa(\hat{\eta})}{\kappa(\hat{\eta})} \cdot \frac{P_V^m}{P_V^c}\right) \int_{\hat{\eta}}^{\eta_1} \eta dF(\eta) - \omega \hat{\eta} F(\hat{\eta}). \tag{S.25}$$

Note that h is continuous, $h(\eta_1) = -\omega\eta_1 < 0$ and

$$h(\hat{\eta}_B) = \frac{1-\kappa(\hat{\eta}_B)}{\kappa(\hat{\eta}_B)} \cdot \frac{P_V^m}{P_V^c} \int_{\hat{\eta}_B}^{\eta_1} \eta dF(\eta) > 0. \tag{S.26}$$

Hence, by the intermediate value theorem, $\hat{\eta} \in (\hat{\eta}_B, \eta_1)$. To complete the proof, note that R is decreasing in $\hat{\eta}$ and, therefore, rent-seeking is reduced compared to the benchmark. ■