Environmental macroeconomics with multiple equilibria

(a) Deterministic models in continuous time

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Outline

1. Motivation & Model
   - Motivation
   - Model: ecological system
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   - Policy tradeoff and welfare analysis
3. Extensions & Conclusions
   - Endogenous labour supply
   - Finite lives
   - Conclusions

Overview & Motivation (1)

- This literature studies an important theme in environmental macroeconomics, namely the effects economic policies (such as public abatement or pollution taxation) on the macroeconomic system and environmental quality
- Issue typically studied in "linear models" in which gradual changes in dirt emissions have gradual effects on the ecological system. Bovenberg & Heijdra (1998, 2002) are examples of this approach
- Here is the 1 million euro question: Does nature respond smoothly to gradual change?
- Around the turn of the century prominent ecologists have answered this question with a resounding NO!

Overview & Motivation (2)

- Scheffer *et al.* (2001) paper in *Nature*, entitled "Catastrophic shifts in ecosystems" presents the (then) new view
  - ecosystems do not respond smoothly to gradual changes in dirt flows
  - abrupt "catastrophic shifts" may be possible in the vicinity of threshold points (no early warning signals)
  - multiple stable equilibria, irreversibility, and hysteresis are all possible
- The prototypical example of the phenomenon under consideration concerns shallow lakes

A literal shallow lake

- "One of the best-studied and most **dramatic state shifts** is the sudden loss of transparency and vegetation observed in shallow lakes subject to human-induced eutrophication. The pristine state of most shallow lakes is probably one of clear water and a rich submerged vegetation. Nutrient loading has changed this situation in many cases. Remarkably, water clarity often seems to be hardly affected by increased nutrient concentration until a **critical threshold** is passed, at which the lake shifts abruptly from clear to turbid. With this increase in turbidity, submerged plants largely disappear. Associated loss of animal diversity and reduction of the high algal biomass makes this state **undesired**. Reduction of nutrient concentrations is often insufficient to restore the vegetated clear state. Indeed, the **restoration of clear water happens at substantially lower nutrient levels than those at which the collapse of the vegetation occurred**." [colored emphasis added]

  Scheffer et al. (2001, p. 592)

Overview & Motivation (3)

- The ecological dynamics under consideration now carries the name **Shallow-Lake Dynamics** (SLD)
- Objective of the first hour today: to study the effects of public abatement activities on the environment and on the economic system when the former system features SLD
- Mode of attack:
  - Basic representative-agent model of a closed economy
  - Ecology features SLD
  - Model solutions explained analytically
  - Non-linear system of differential equations is solved numerically
  - Size versus duration tradeoff
  - Welfare analysis: first-best versus second-best
  - Robustness check: alternative model assumptions

Pollution dynamics

- Flow of dirt:

$$D(t) \equiv \kappa K(t) - \gamma G(t), \qquad \kappa > 0,\ \gamma > 0 \qquad \text{(S1)}$$

  - $K(t)$ is the capital stock
  - $G(t)$ is government abatement
  - $D(t)$ is the flow of dirt; feasibility constraint: $D(t) \geq 0$

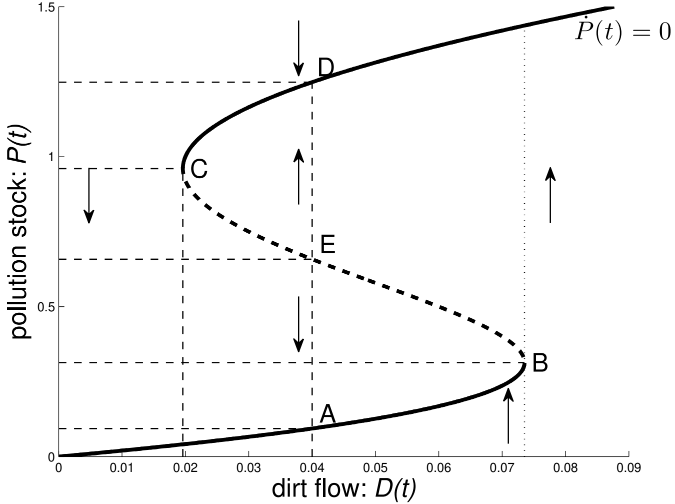
- Dynamic evolution of the pollution stock:

$$\dot{P}(t) = -\pi P(t) + \frac{P(t)^2}{P(t)^2 + 1} + D(t), \qquad \frac{1}{2} < \pi < \frac{3\sqrt{3}}{8} \qquad \text{(S2)}$$

  - first term on the RHS of (S2): stable regeneration
  - second term on the RHS of (S2): shallow-lake dynamics (SLD)
  - $\pi > \frac{1}{2}$ implies that nature contains **no irreversibilities**

- See **Figure 1** for dynamics of $P(t)$

Figure 1: Ecological dynamics



Key features of Figure 1

- threshold points D_L (below C) and D_U (below B)
- dashed branch between C and B unstable
- two stable branches (solid lines)
- for $\hat{D}_0$ two, welfare-rankable, stable equilibria (D and A)
- ecological system features **reversible hysteresis**: A temporary dirt reduction can move the ecology from D to A

Households (1)

- Infinitely-lived representative agent
- Lifetime utility function:

$$\Lambda(t) \equiv \int_t^\infty \left[\ln C(\tau) + \varepsilon_E \ln \left(\bar{E} - P(\tau) \right) \right] e^{-\rho(\tau - t)} d\tau \quad \text{(S3)}$$

  - $C(\tau)$ is consumption
  - $E(\tau) \equiv \bar{E} - P(\tau)$ measures the quality of the environment
  - ε_E is the utility weight of environmental quality
  - ρ is the pure rate of time preference
- Household budget identity:

$$\dot{A}(\tau) = r(\tau)A(\tau) + w(\tau) - T(\tau) - C(\tau) \quad \text{(S4)}$$

  - $A(\tau)$ is financial assets ($\dot{A}(\tau) \equiv dA(\tau)/d\tau$)
  - $r(\tau)$ is the interest rate
  - $w(\tau)$ is the wage rate
  - $T(\tau)$ is the lump-sum tax

Households (2)

- Labour supply is exogenous and equal to unity, so $w(\tau)$ also stands for the household's wage income
- At time t, the agent chooses time paths for $C(\tau)$ and $A(\tau)$ $(\tau \geq t)$ in order to maximize (S3) subject to (S4), the initial condition on assets, $A(t)$, and the TVC
- Solutions:

$$C(t) = \rho[A(t) + H(t)] \tag{S5}$$

$$H(t) \equiv \int_t^{\infty} [w(\tau) - T(\tau)]\, e^{-\int_t^{\tau} r(s)ds} d\tau \tag{S6}$$

$$\frac{\dot{C}(\tau)}{C(\tau)} = r(\tau) - \rho, \qquad \tau \geq t \tag{S7}$$

Firms (1)

- Many, perfectly competitive firms (CRTS)
- Representative firm
- Production function:

$$Y(t) \equiv F(K(t), L(t)) = \Omega_0 K(t)^{\varepsilon_L} L(t)^{1-\varepsilon_L} \tag{S8}$$

  - $Y(t)$ is gross output
  - $L(t)$ is employment
  - Parameters: $\Omega_0 > 0,\ 0 < \varepsilon_L < 1$
- Value of the firm:

$$V(t) = \int_t^{\infty} \left[(1 - \theta_K(\tau))\Big(Y(\tau) - w(\tau)L(\tau)\Big) - I(\tau)\right] e^{-\int_t^{\tau} r(s)ds} d\tau \tag{S9}$$

Firms (2)

- Capital accumulation identity:

$$\dot{K}(\tau) = I(\tau) - \delta K(\tau) \tag{S10}$$

- First-order conditions:

$$w(\tau) = F_L\left(K(\tau), L(\tau)\right) \tag{S11a}$$

$$\frac{r(\tau) + \delta}{1 - \theta_K(\tau)} = F_K\left(K(\tau), L(\tau)\right) \tag{S11b}$$

- It follows that $V(t) = K(t)$ (no adjustment costs of investment so Tobin's q is equal to unity)

General equilibrium

- Model summarized in **Table 1**
- Exogenous labour supply: $L^S(t) = 1$
- Government policies:
  - No capital taxation: $\theta_K(t) = 0$
  - Government abatement activities: $G(t)$
  - Lump-sum tax balances the budget: $G(t) = T(t)$
- Endogenous variables: $C(t)$, $Y(t)$, $K(t)$, $P(t)$, $w(t)$, $r(t)$, $L(t)$, and $D(t)$
- Exogenous variable: $G(t)$
- **Figure 2**. Saddle-point stable, dynamically efficient, unique equilibrium at E_0

Table 1: The benchmark continuous-time model

(a) Economic system

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho, \qquad \rho > 0 \tag{T1.1}$$

$$\dot{K}(t) = Y(t) - C(t) - G(t) - \delta K(t) \tag{T1.2}$$

$$[r(t) + \delta]\, K(t) = (1 - \varepsilon_L) Y(t) \tag{T1.3}$$

$$w(t)L(t) = \varepsilon_L Y(t) \tag{T1.4}$$

$$Y(t) = \Omega_0 L(t)^{\varepsilon_L} K(t)^{1-\varepsilon_L} \tag{T1.5}$$

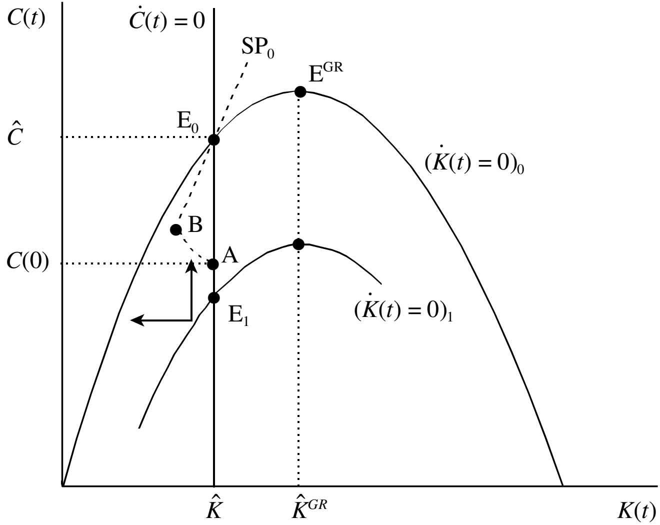
$$L(t) = 1 \tag{T1.6}$$

(b) Ecological system

$$\dot{P}(t) = -\pi P(t) + \frac{P(t)^2}{P(t)^2 + 1} + D(t) \tag{T1.7}$$

$$D(t) = \kappa K(t) - \gamma G(t), \qquad \kappa > 0,\ \gamma > 0 \tag{T1.8}$$

Figure 2: C-K dynamics in the core model



Temporary abatement shock (1)

- Initially at a steady-state equilibrium E_0
- No abatement initially, $G(t) = 0$ for $t < 0$
- At time $t = 0$, an unanticipated and temporary increase in $G(t)$:

$$G(t) = \begin{cases} G & \text{for } 0 \leq t \leq t_E \\ 0 & \text{for } t > t_E \end{cases} \quad \text{(S12)}$$

- Qualitative adjustment path in **Figure 2**:
  - jump from E_0 to A at impact $(t = 0)$
  - for $0 < t \leq t_E$, gradual adjustment from A to B
  - arrive at B at time $t = t_E$
  - for $t_E < t \leq \infty$, gradual adjustment from B to E_0

Temporary abatement shock (2)

- Visualization of all adjustment paths in **Figures 3(a)–(f)**
- Economic parameters: $\rho = \hat{r} = 0.04$, $\delta = 0.07$, $\varepsilon_L = 0.7$, $\Omega_0 = 0.7401$, and $\theta_K = 0$
- Steady-state features: $\hat{K}/\hat{Y} = 2.7273$, $\hat{Y} = 1$, $\hat{I} = \delta\hat{K} = 0.1909$, $\hat{C} = 0.8091$
- Ecological parameters: $\pi = 0.52$, $\kappa = 0.0147$, and $\gamma = 0.3020$
- Steady-state features: $D_L = 0.0196$, $D_U = 0.0735$, $\hat{D}_0 = \kappa\hat{K} = 0.04$, two stable ecological equilibria: points A (with $\hat{P}_G = 0.0936$) and D (with $\hat{P}_B = 1.2482$) in Figure 3(b). The critical pollution stock associated with $\hat{P}_0$ is at point E where $P_E = 0.6581$
- For $G = 0.1$ the minimum shock duration is $t_E = 41$ years

Figure 3(a): government abatement $G(t)$

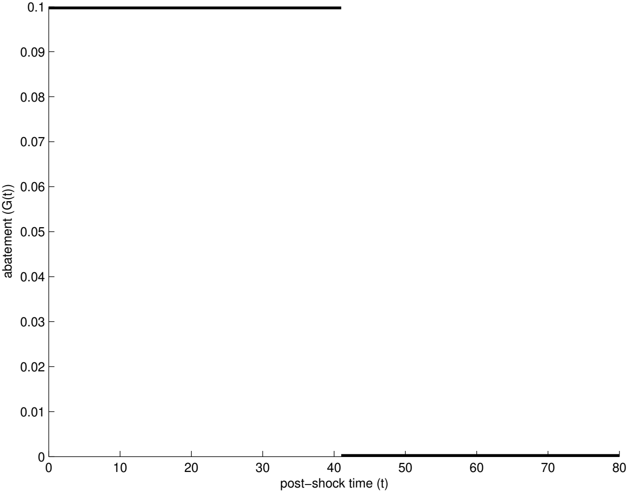


Figure 3(b): pollution dynamics $\dot{P}(t)$

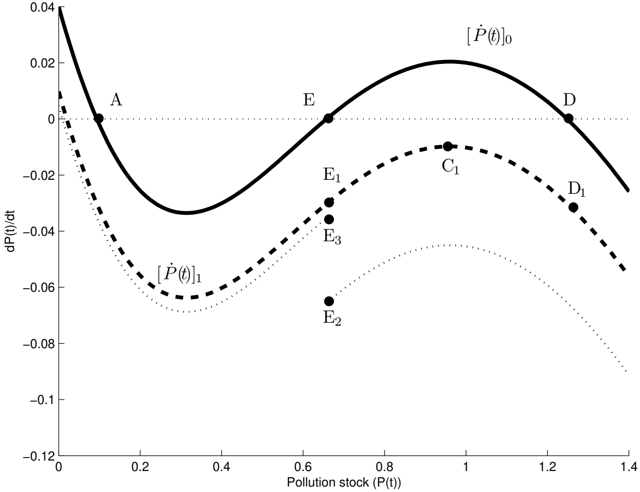


Figure 3(c): capital stock $K(t)$

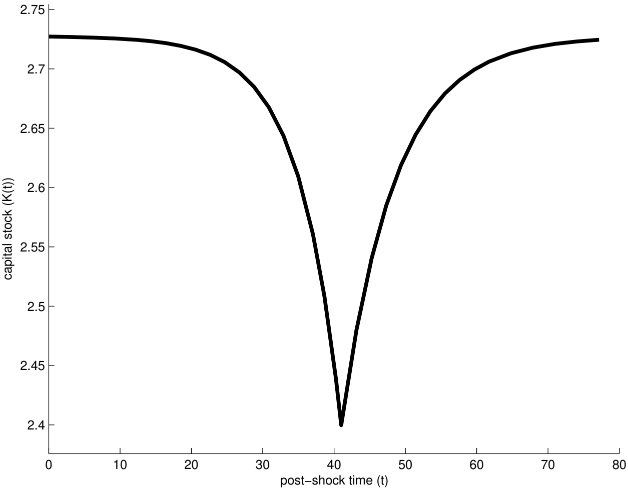


Figure 3(d): consumption $C(t)$

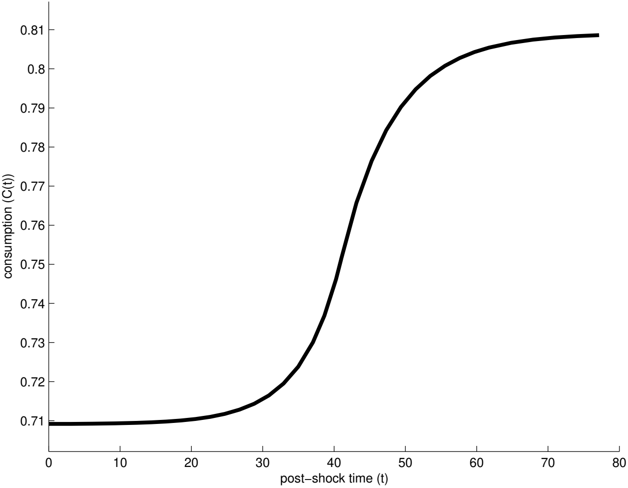


Figure 3(e): dirt flow $D(t)$

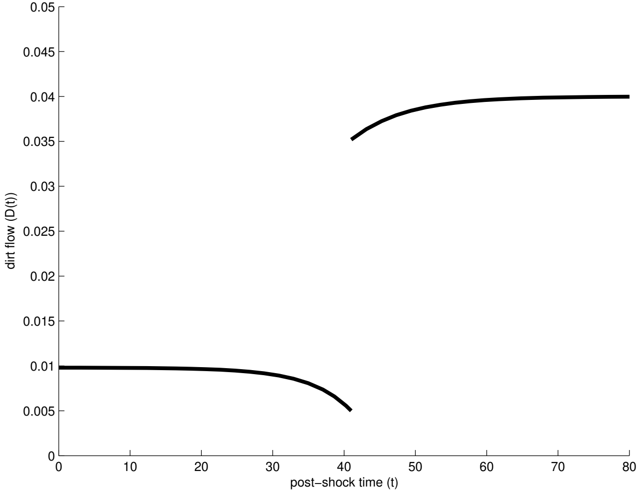
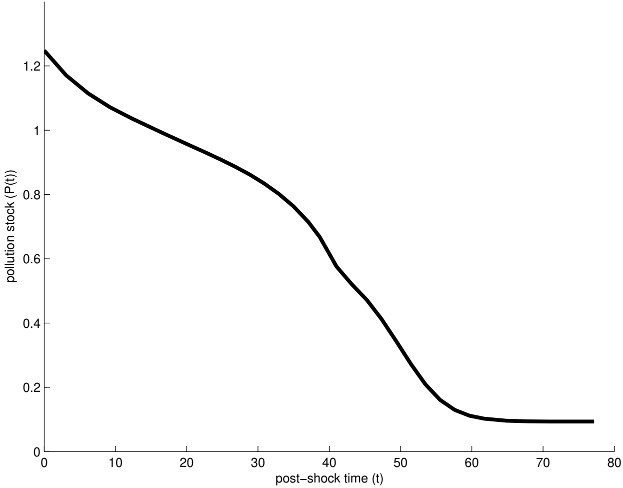


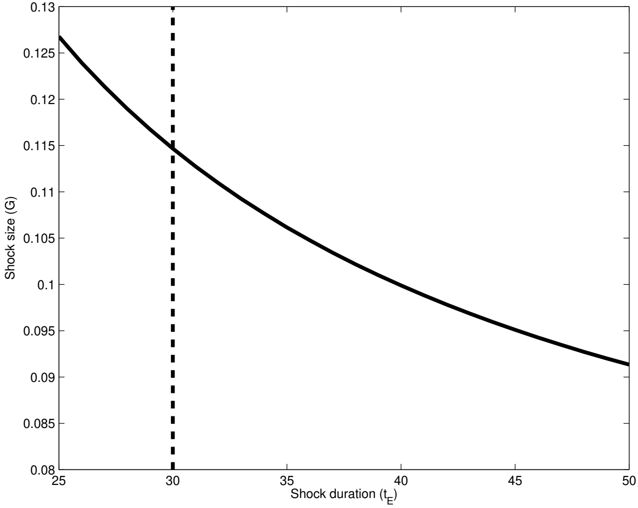
Figure 3(f): pollution stock $P(t)$



Size-duration tradeoff

- A larger abatement shock can be maintained for a briefer period (and yet be succesful) because adjustment is faster
- Tradeoff between G and t_E analyzed in **Figure 4(a)**
    - region to the left of the dashed line is infeasible ($D \geq 0$ violated for some t during transition)
    - region to the right of the dashed line is feasible ($D \geq 0$ satisfied for all t during transition)
    - tradeoff is downward sloping
- Which (G, t_E) combination is optimal?

Figure 4(a): size G versus duration t_E



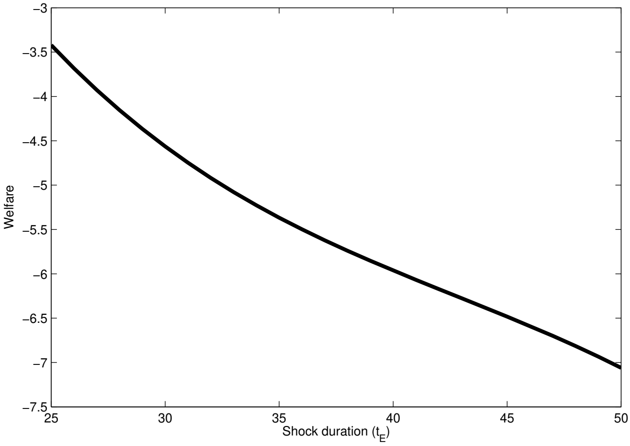
Welfare effects (1)

- Two welfare questions
- **Question 1: which size-duration (G, t_E) combination is optimal?**

▷ **Figure 4(b)** plots optimized value of welfare $\Lambda(0)$ as a function of t_E (with associated G value taken from the size-duration tradeoff schedule)

▷ "Cold-turkey" (or "big bang") policy is best: minimum feasible duration, maximum size (corner of the feasible region in Figure 4(a))

Figure 4(b): welfare effect $\Lambda(0)$



Welfare effects (2)

- **Question 2: starting in the dirty equilibrium at point D (in Figure 1), where on the lower branch should the equilibrium be moved to?**
- *First-best equilibrium*: move to the left of point A (lower steady-state dirt flow). Internalize external pollution effect of capital accumulation
  - Decentralized with $G = 0$ in the long run and a Pigouvian capital tax: $\theta_K = \frac{\kappa}{\gamma(\rho+\delta)+\kappa}$
- *Second-best equilibrium*: in the absence of a Pigouvian tax instrument FBSO cannot be decentralized as privately optimal savings behaviour leads to an equalization of the net marginal product of capital to the rate of time preference, so that the capital stock is equal to $\hat{K}$ $(> \hat{K}_f)$. Move to point A
  - SBSO features $G = 0$ in the long run. Hence, move from D to A in **Figure 1** as fast as possible

Overview of extensions

- Endogenous labour supply ▸ Skip details
  - endogenous hours decision
  - tax-financed abatement policy increases both labour supply and the capital stock for part of the transition period
  - this dirties the environment and makes it harder to steer the economy from the dirty to the clean equilibrium
- Finite lives and overlapping generations ▸ Skip details
  - exogenous labour supply
  - crowding out of capital during transition facilitates the environmental cleanup
  - intergenerational redistribution effects during transition
  - role for debt policy: redistribution
  - effect of debt policy: hysteresis in the economic system

Endogenous labour supply (1)

- Change utility function:

$$\Lambda(t) \equiv \int_t^{\infty} \Big[\ln \Big(C(\tau)^{\varepsilon_C} \left[1 - L(\tau)\right]^{1-\varepsilon_C} \Big) + \varepsilon_E \ln \left(\bar{E} - P\left(\tau\right) \right) \Big] e^{-\rho(\tau - t)} d\tau \qquad \text{(S13)}$$

- Change budget identity:

$$\dot{A}\left(\tau\right) = r(\tau)A(\tau) + w(\tau)L\left(\tau\right) - T(\tau) - C(\tau) \qquad \text{(S14)}$$

- Optimal decision rules:

$$C(t) = \rho\varepsilon_C[A(t) + H(t)] \qquad \text{(S15a)}$$

$$w(t)\left[1 - L(t)\right] = \rho\left(1 - \varepsilon_C\right)\left[A(t) + H(t)\right] \qquad \text{(S15b)}$$

Endogenous labour supply (2)

- Labour supply equation (replaces (T1.6) in **Table 1**):

$$w(t)\,[1 - L(t)] = \frac{1 - \varepsilon_C}{\varepsilon_C} C(t) \qquad \text{(S16)}$$

- Phase diagram in **Figure 5**
- Visualization in **Figures 6(a)–(f)**
- For $G = 0.1$ we now need $t_E = 52$ (rather than $t_E = 41$) to get from D to A. Labour supply effect makes environmental policy more difficult
- FBSO and SBSO both call for $G = 0$ in the long run

Figure 5: C-K dynamics with endogenous labour supply

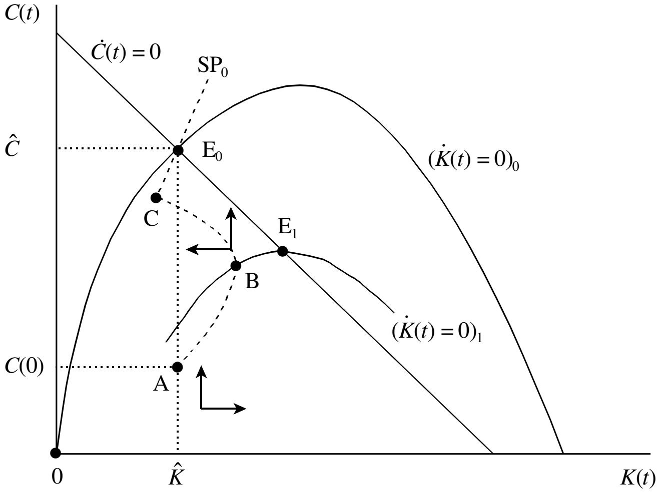


Figure 6(a): capital stock $K(t)$

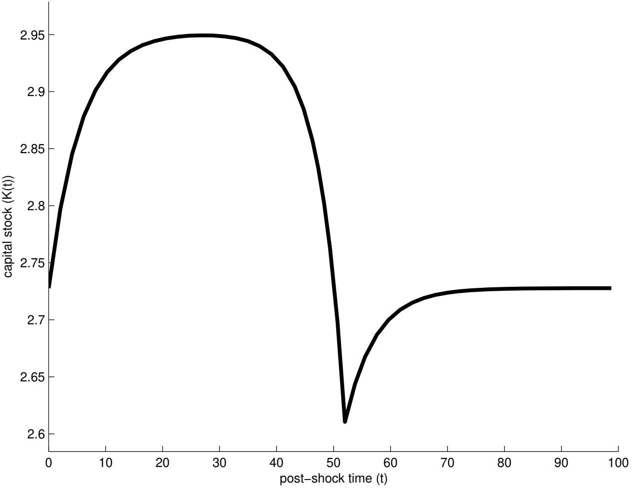


Figure 6(b): consumption $C(t)$

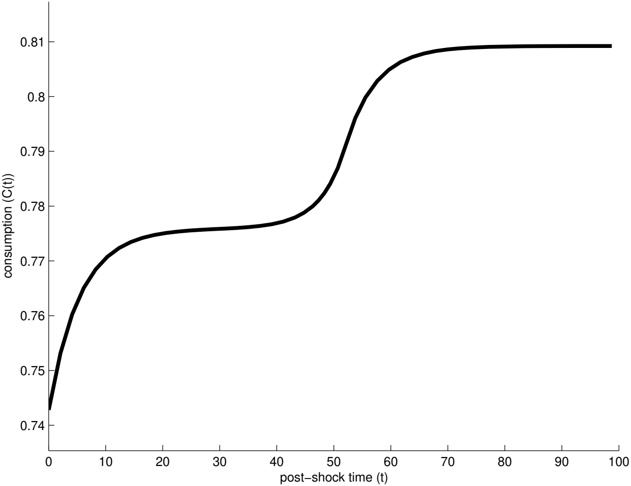


Figure 6(c): output $Y(t)$

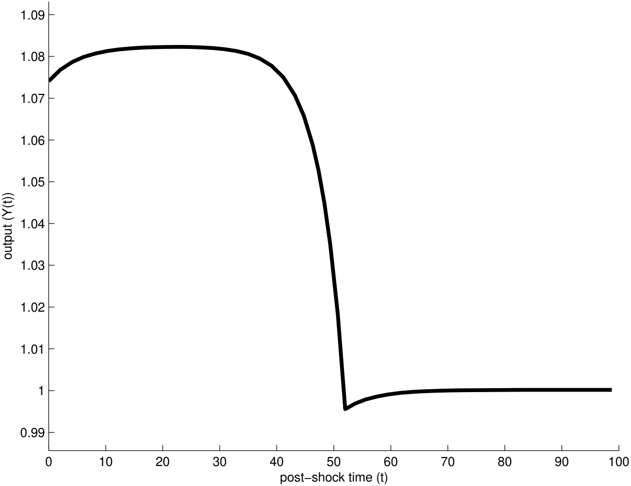


Figure 6(d): employment $L(t)$

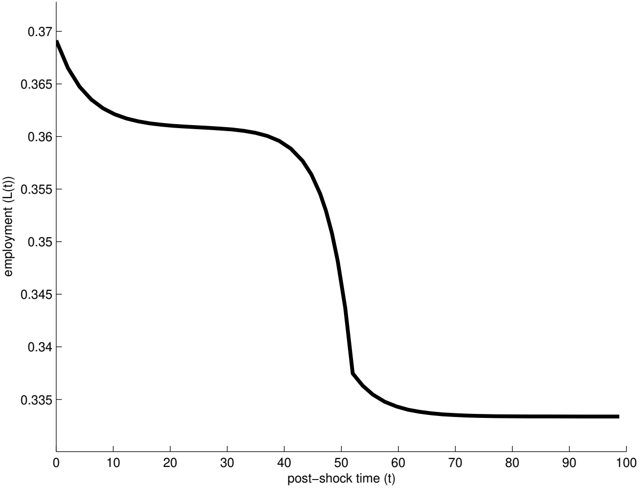


Figure 6(e): dirt flow $D(t)$

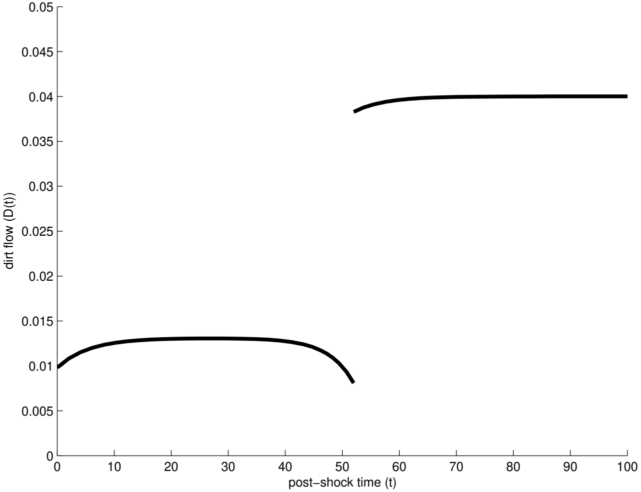
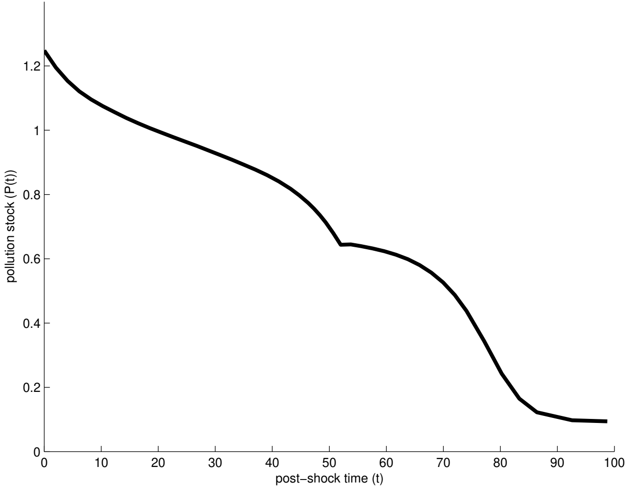


Figure 6(f): pollution stock $P(t)$



Overlapping generations (1)

Skip details

- Perpetual-youth model: constant instantaneous probability of death, $\mu > 0$
- Individual lifetime utility:

$$\mathbb{E}\Lambda(v,t) \equiv \int_t^{\infty} \left[\ln C(v,\tau) + \varepsilon_E \ln\left(\bar{E} - P(\tau)\right) \right] e^{-(\rho+\mu)(\tau-t)} d\tau \quad \text{(S17)}$$

- Budget identity under full annuitization:

$$\dot{A}(v,\tau) = [r(\tau) + \mu] A(v,\tau) + w(\tau) - C(v,\tau) - T(\tau) \quad \text{(S18)}$$

  - $r(\tau) + \mu$ is the rate of return on annuities
  - agents born bare of assets, $A(v,v) = 0$

Overlapping generations (2)

- Individual decision rules at time t:

$$C(v,t) = (\rho+\mu)\left[A(v,t)+H(t)\right] \tag{S19}$$

$$H(t) \equiv \int_t^{\infty} \left[w(\tau)-T(\tau)\right] e^{-\int_t^{\tau}[r(s)+\mu]ds} d\tau \tag{S20}$$

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = r(\tau)-\rho, \qquad \tau \geq t \geq v \tag{S21}$$

- Aggregate implication for the "Euler equation" (replaces (T1.1) in **Table 1**):

$$\frac{\dot{C}(t)}{C(t)} = r(t)-\rho-\mu(\rho+\mu)\frac{K(t)+B(t)}{C(t)} \tag{S22}$$

- Phase diagram (for $B(t)=0$) in **Figure 7**
- Visualization in **Figures 8(a)–(f)**

Figure 7: C-K dynamics with overlapping generations

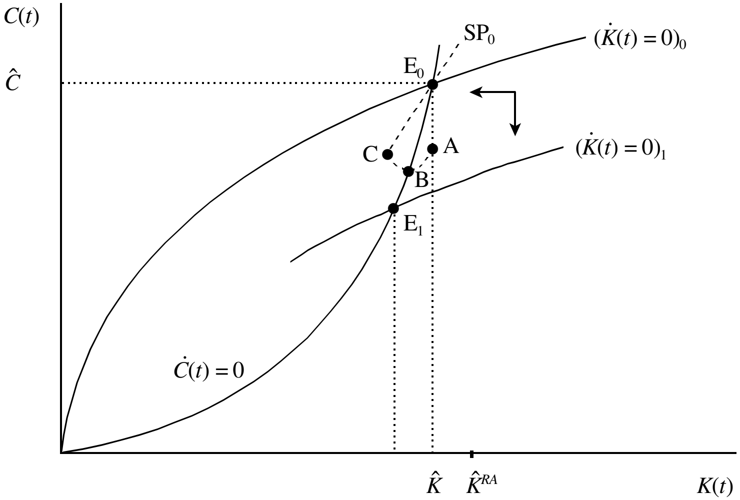


Figure 8(a): consumption $C(t)$

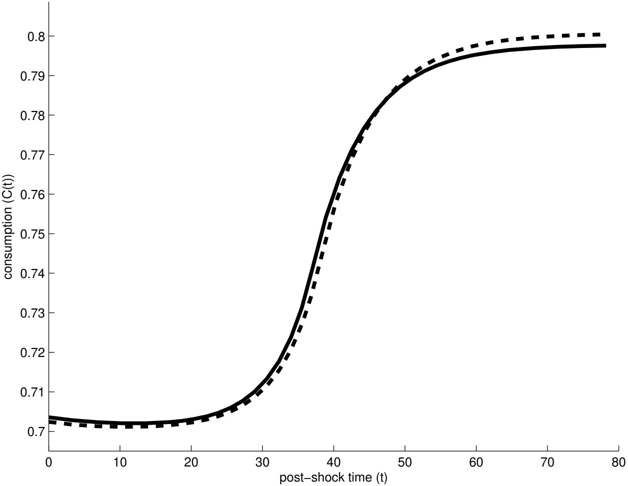


Figure 8(b): capital stock $K(t)$

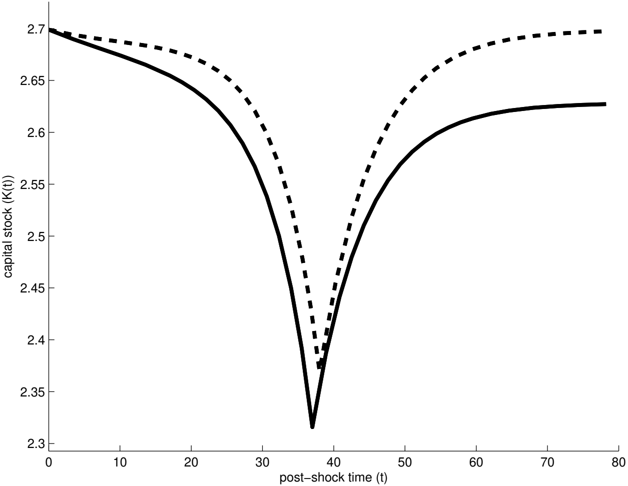


Figure 8(c): government abatement $G(t)$ and taxes $T(t)$

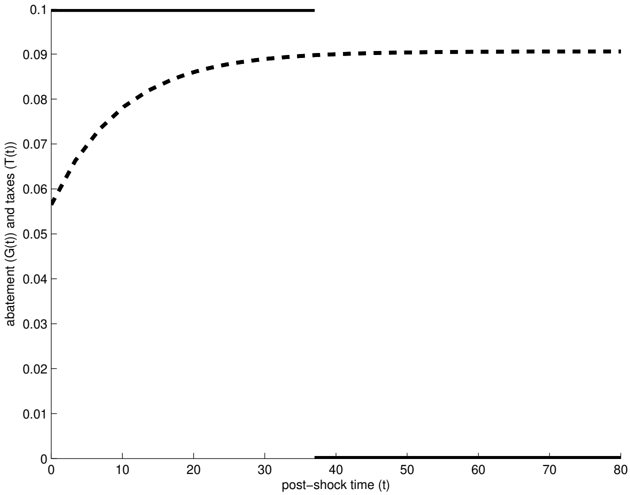


Figure 8(d): debt-output ratio $B(t)/Y(t)$

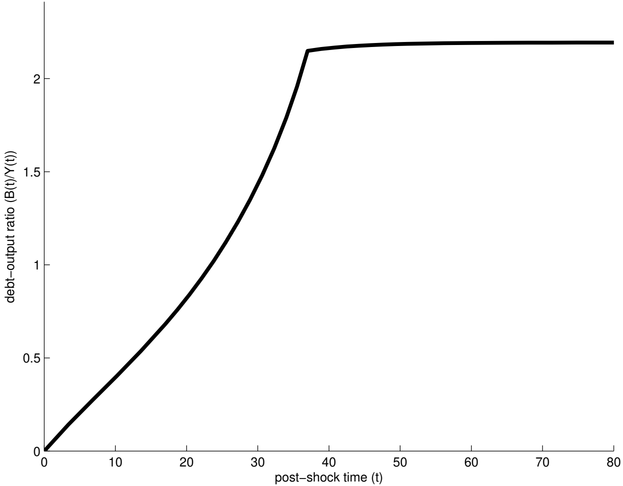


Figure 8(e): dirt flow $D(t)$

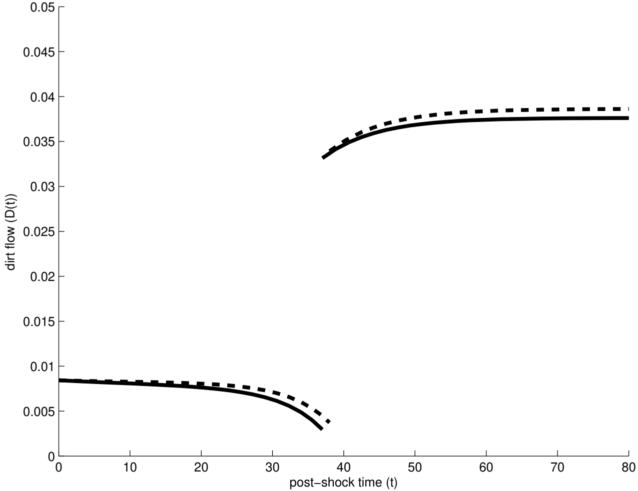
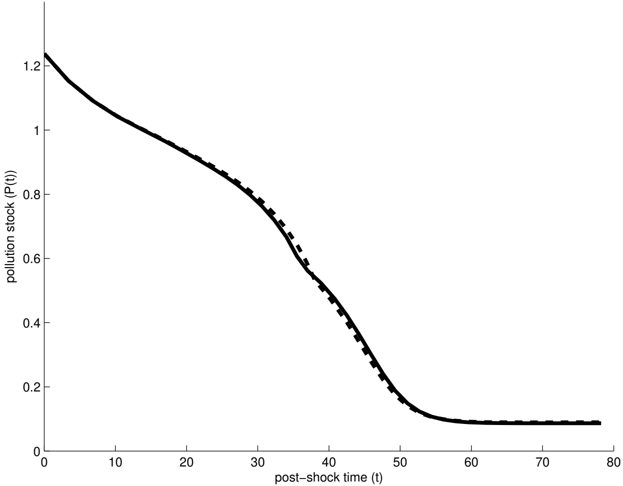


Figure 8(f): pollution stock $P(t)$



Overlapping generations (3)

- Government solvency condition:

$$B(t) = \int_t^\infty [T(\tau) - G(\tau)] e^{-\int_t^\tau r(s)ds} d\tau \qquad \text{(S23)}$$

- The bond policy that we consider takes the following form:
  - Debt is zero initially, i.e. $B(0) = 0$
  - Parametric tax path of the form $T(t) = T_0 + T_1 \left[1 - e^{-\xi t}\right]$ for $t \geq 0$ and $\xi > 0$. Here $T(0) = T_0$ stands for the initial tax, $T(\infty) = T_0 + T_1$ is the long-run tax, and ξ is the speed of debt stabilization
  - Government solvency condition in terms of parameters:

$$\int_0^\infty \left[T_0 + T_1 \left[1 - e^{-\xi t}\right]\right] e^{-\int_t^\tau r(s)ds} d\tau = G \int_0^{t_E} e^{-\int_t^\tau r(s)ds} d\tau \qquad \text{(S24)}$$

Overlapping generations (4)

- An abatement *cum* debt policy consists of the vector (G, t_E, T_0, T_1, ξ) such that (i) t_E is as small as feasible for the given shock, and (ii) equation (S24) is satisfied by suitable choice of T_0 and/or T_1
- Without debt policy: $G = 0.1$ requires $t_E = 38$ (slightly quicker than core case, for which $t_E = 41$)
- With debt policy: $G = 0.1$ ($T_0 = 0.0616$, $T_1 = 0.0290$, $\xi = 0.1$) requires $t_E = 37$ (slightly quicker than core case). Small amount of crowding out of the private capital stock by debt

Main findings

- A proper public abatement policy takes a radically different form than was previously thought if the ecology features shallow-lake dynamics
- A temporary policy may succeed in getting from the bad to the good equilibrium. Abatement is an effective instrument for that
- There is a tradeoff between shock size and shock duration
- From a second-best welfare perspective, a “cold-turkey policy” is best
- From a first-best welfare perspective, abatement should be used temporarily and a capital tax should internalize the external effects due to capital accumulation
- Endogenous labour supply complicates policy, whilst finite lives facilitate policy

Literature

- Bovenberg, A. L. and Heijdra, B. J. (1998). Environmental tax policy and intergenerational distribution. *Journal of Public Economics*, 67:1–24.
- Bovenberg, A. L. and Heijdra, B. J. (2002). Environmental abatement and intergenerational distribution. *Environmental and Resource Economics*, 23:45–84.
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Literature

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