Foundations of Modern Macroeconomics
Second Edition

Chapter 14: Endogenous economic growth (sections 14.4 – 14.5)

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Outline

R & D as the engine of growth

- *Key idea*: Purposeful conduct of R&D activities is the source of growth.
  - In the absence of human and physical capital, households can nevertheless save by accumulating *patents*.
  - Patents are blueprints for the production of "slightly unique" products.
  - The patent holder has a little bit of monopoly power which can be exploited.
  - Hence, in this literature we leave the competitive framework and enter the realm of monopolistic competition. (Schumpeterian models of "creative destruction" can be built along the lines of the present model. Example: RIQ model.)

Overview of the expanding input variety (EIV) model

- Three productive sectors.
- *Final goods sector* (CRTS, perfectly competitive, external effect "returns to specialization"): Produces a homogenous good using differentiated inputs in the production process.
- *Intermediate goods sector* (many small monopolistically competitive firms): Each firm (patent holder) uses labour to produce its own slightly unique variety of the intermediate input.
- *R&D sector* (CRTS, perfectly competitive, external effect "standing on the shoulder of giants"): Produces blueprints for new intermediate inputs, using labour as an input.
- Production factors perfectly mobile.

Final goods sector (1)

- *Final goods sector* (CRTS, perfectly competitive, external effect "returns to specialization"): Produces a homogenous good using differentiated inputs in the production process.
- Technology:

$$Y(t) \equiv N(t)^{\eta} \cdot \left[\frac{1}{N(t)} \sum_{i=1}^{N(t)} X_i(t)^{1/\mu} \right]^{\mu} \qquad \text{(S1)}$$

  where X_i is intermediate input i, N is the existing number of varieties, and μ and η are parameters ($\mu > 1$ and $\eta \geq 1$).
- If $\eta > 1$ there are returns to specialization as in Adam Smith's pin factory. If intermediate inputs are more finely differentiated then firms can use a more roundabout production process.
- μ measures the ease with which inputs can be substituted. This is the source of market power later on.

Final goods sector (2)

- Pricing decision:

$$P_Y(t) \equiv N(t)^{-\eta} \cdot \left[N(t)^{\mu/(1-\mu)} \sum_{i=1}^{N(t)} P_i(t)^{1/(1-\mu)} \right]^{1-\mu}$$

- Derived demand for input i (for $(i = 1, 2, \cdots, N(t))$):

$$\frac{X_i(t)}{Y(t)} = N(t)^{(\eta-\mu)/(\mu-1)} \cdot \left(\frac{P_i(t)}{P_Y(t)} \right)^{\mu/(1-\mu)}$$

So $\mu/(1-\mu)$ is the demand elasticity.

Intermediate goods sector

- *Intermediate goods sector* (many small monopolistically competitive firms): Each firm (patent holder) uses labour to produce its own slightly unique variety of the intermediate input.
- Technology:

$$X_i(t) = Z_X \cdot L_i(t)$$

  constant marginal production costs.
- Pricing decision:

$$P_i(t) = \mu \cdot \frac{W(t)}{Z_X}$$

  where μ is the gross monopoly markup.

R & D sector

- *R&D sector* (CRTS, perfectly competitive, external effect "standing on the shoulder of giants"): Produces blueprints for new intermediate inputs, using labour as an input.
- Technology:

$$\dot{N}(t) = Z_R \cdot N(t) \cdot L_R(t)$$

  Labour engaged in the R&D sector becomes more productive as more patents already exist. Today's engineers "stand on the shoulders of giants."
- Pricing decision:

$$P_N(t) = \frac{(1 - s_R) \cdot W(t)}{Z_R \cdot N(t)}$$

Household behaviour

- Representative infinitely-lived households.
- Lifetime utility function:

$$\Lambda(0) = \int_0^\infty \frac{C(t)^{1-1/\sigma} - 1}{1 - 1/\sigma} \cdot e^{-\rho t} dt$$

- Household budget identity:

$$P_Y(t)C(t) + P_N(t)\dot{N}(t) = W(t)L_0 - T(t) + N(t)\bar{\Pi}(t)$$

- Optimality conditions:

$$\frac{\dot{C}(t)}{C(t)} = \sigma \cdot [r(t) - \rho]$$

$$r(t) = \frac{\bar{\Pi}(t) + \dot{P}_N(t)}{P_N(t)}$$

where $r(t)$ is the rate of return on blueprints.

Loose ends

- Final goods market:

$$Y(t) = C(t) + G(t), \qquad G(t) = gY(t)$$

where g is a policy variable.

- Government budget constraint:

$$T(t) = G(t) + s_R W(t) L_R(t)$$

- Labour market equilibrium:

$$L_X(t) + L_R(t) = L_0$$

- But $Z_X L_X(t) = N(t)\bar{X}(t)$ and $Z_R L_R(t) = \dot{N}(t)/N(t)$ so:

$$\frac{\dot{N}(t)}{N(t)} = Z_R \left[L_0 - \frac{N(t)\bar{X}(t)}{Z_X} \right] > 0$$

Assumption: intermediate sector does not absorb the entire labour force.

Solving the model (1)

- *Step 1*. Intermediate results:

$$
\begin{aligned}
\frac{\bar{\Pi}(t)}{P_N(t)} &= (\mu-1)\frac{Z_R}{1-s_R}L_X(t) \\
\frac{\dot{P}_N(t)}{P_N(t)} &= (\eta-2)\frac{\dot{N}(t)}{N(t)} \\
C(t) &= (1-g)\,N(t)^{\eta-1}Z_X L_X(t)
\end{aligned}
$$

- *Step 2*. Dynamic equations:

$$
\begin{aligned}
\gamma_C(t) &= \sigma\left[(\mu-1)\,\frac{Z_R}{1-s_R}L_X(t)+(\eta-2)\gamma_N(t)-\rho\right] \\
\gamma_C(t) &= (\eta-1)\gamma_N(t)+\frac{\dot{L}_X(t)}{L_X(t)} \\
\gamma_N(t) &= Z_R\left[L_0-L_X(t)\right]
\end{aligned}
$$

Solving the model (2)

- *Step 3.* Combine dynamic equations to get:

$$\frac{\dot{L}_X(t)}{L_X(t)} = Z_R \cdot \left[\frac{\sigma(\mu - 1)}{1 - s_R} + \eta - 1 + \sigma(2 - \eta)\right](L_X(t) - L_X^*) \tag{A}$$

where L_X^* is defined as:

$$L_X^* = \frac{[\eta - 1 + \sigma(2 - \eta)] L_0 + \sigma\rho / Z_R}{\sigma(\mu - 1) / (1 - s_R) + \eta - 1 + \sigma(2 - \eta)}$$

- Eq. (A) is an unstable differential equation in $L_X(t)$. Only economically sensible solution is $L_X(t) = L_X^*$.
- Hence $\gamma_N(t) = \gamma_N^* \equiv Z_R[L_0 - L_X^*]$ is also time-invariant. There is no transitional dynamics (no capital).

Economic growth

- The growth rates are:

$$\gamma_N^* = \frac{\frac{\mu-1}{1-s_R} Z_R L_0 - \rho}{\frac{\mu-1}{1-s_R} + \frac{\eta-1}{\sigma} + (2-\eta)} > 0$$

$$\gamma_C^* = \gamma_Y^* = (\eta - 1)\gamma_N^*$$

- The innovation rate, γ_N^*:
  - increases with the monopoly markup (μ) and the subsidy (s_R)
  - increases with the size of the labour force (L_0)
  - (provided $\eta > 1$) increases with the intertemporal substitution elasticity (σ)
  - decreases with the rate of time preference (ρ).
- Consumption and aggregate output grow only if the returns to specialization are operative (so that $\eta > 1$).

Efficiency (1)

- Not obvious that the decentralized market equilibrium is efficient as there are both external effects and non-competitive behaviour.
- "Quick-and-dirty" intuition would seem to suggest that there is too little innovation (under-investment in R&D) because the innovator does not capture all the beneficial effects of his act.
- Formal approach in economic theory:
  - Compute what kind of allocation a (benevolent) social planner would choose
  - Such a planner takes into account ("internalizes") all external effects/economies of scale
  - Compare socially optimal allocation with decentralized market allocation
  - How can social optimum be replicated in the market?

Efficiency (2): The social optimum

- Social planner imposes symmetry up front and works directly with aggregates.
- Current-value Hamiltonian:

$$\mathcal{H}_C(t) = \frac{\left[N(t)^{\eta-1} Z_X L_X(t)\right]^{1-1/\sigma} - 1}{1-1/\sigma} + \mu_N(t) N(t) Z_R \left[L_0 - L_X(t)\right]$$

where $\mu_N(t)$ is the co-state variable for $N(t)$.

- FONCs:
  - For $L_X(t)$:

$$Z_R \mu_N(t) = \frac{Z_X N(t)^{\eta-2}}{\left[N(t)^{\eta-1} Z_X L_X(t)\right]^{1/\sigma}}$$

  - For $N(t)$:

$$\dot{\mu}_N(t) = \rho \mu_N(t) - \frac{(\eta-1) Z_X L_X(t) N(t)^{\eta-2}}{\left[N(t)^{\eta-1} Z_X L_X(t)\right]^{1/\sigma}} - \mu_N(t) Z_R \left[L_0 - L_X(t)\right]$$

Efficiency (3): The social optimum

- Combine the FONCs:

$$\frac{\dot{L}_X(t)}{L_X(t)} = (\eta - 1)\, Z_R L_X(t) - (\eta - 1)(1 - \sigma) Z_R L_0 - \sigma \rho$$

- Provided $\eta > 0$ this is an unstable differential. Optimal solution is time invariant:

$$L_X^{SO} = (1 - \sigma) L_0 + \frac{\sigma \rho}{(\eta - 1)\, Z_R}$$

- Optimal innovation rate:

$$\begin{aligned}
\gamma_N^{SO} &\equiv Z_R \left[L_0 - L_X^{SO} \right] = \sigma Z_R L_0 - \frac{\sigma \rho}{\eta - 1} > 0 \\
\gamma_C^{SO} &= \gamma_Y^{SO} = (\eta - 1) \gamma_N^{SO}
\end{aligned}$$

Efficiency (4): The comparison

- (For simple case, $\sigma = 1$) we find:

$$\gamma_N^*(s_R) \equiv \frac{(\mu-1)Z_R L_0 - \rho(1-s_R)}{\mu - s_R}$$

$$\gamma_N^{SO} = \frac{(\eta-1)Z_R L_0 - \rho}{\eta - 1}$$

- Point of view #1: Suppose $s_R = 0$.

$$\mu \cdot \left[\gamma_N^{SO} - \gamma_N^*(0)\right] = Z_R L_0 - \rho \cdot \frac{\mu - (\eta - 1)}{\eta - 1}$$

  - no general conclusion. $\gamma_N^{SO} > \gamma_N^*(0)$ or $\gamma_N^{SO} < \gamma_N^*(0)$?
  - if $\eta = \mu$ (knife-edge case): Q&D intuition is OK! We find that $\gamma_N^{SO} > \gamma_N^*(0)$
  - if $\eta \approx 1$ (weak specialization effect): there may be too much innovation!

Efficiency (5)

- Point of view #2: For which subsidy value do we find $\gamma_N^{SO} = \gamma_N^*(s_P)$?

$$\frac{s_R^*}{1-s_R^*} = \frac{(\eta-1) Z_R L_0 - \rho[\mu-(\eta-1)]}{\rho(\mu-1)} = \frac{\mu(\eta-1)}{\rho(\mu-1)} \cdot \left[\gamma_N^{SO} - \gamma_N^*(0)\right]$$

  - it is optimal to subsidize (tax) R&D labour if the *laissez-faire* economy innovates too slowly (quickly) relative to the social optimum.

Counterfactual prediction (1)

- *Problematic aspect*: The growth rate depends on the scale of the economy (L_0 in this case). Hence, large countries should grow faster than small countries. This is not observed in reality. Jones removes the scale effect by replacing the R&D technology by:

$$\dot{N}(t) = Z_R \cdot L_R(t) \cdot N(t)^{\phi_1} \cdot \left[\bar{L}_R(t)\right]^{\phi_2 - 1}$$

  where $\bar{L}_R$ is average R&D labour per R&D firm.
- We had $\phi_1 = 1$ but now assume $0 < \phi_1 < 1$ (the giants don't grow forever).
- We had $\phi_2 = 1$ but now assume $0 < \phi_2 \leq 1$ (duplication externality: individual R&D firms think the production function is linear, but in actuality it features diminishing returns to labour).

Counterfactual prediction (2)

- Assuming that the population grows at an exponential rate, n_L, the growth rates are now (see book for details):

$$\begin{aligned}\gamma_N^* &= \frac{\phi_2 n_L}{1-\phi_1} \\ \gamma_y^* &= \gamma_Y^* - n_L = (\mu - 1)\gamma_N^* \\ \gamma_c^* &= \gamma_C^* - n_L = \gamma_y^*\end{aligned}$$

- We reach the striking conclusion that by eliminating the scale effect we are back in the realm of exogenous growth and the Solow model!