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Joachim Ossip Mierau

Annuities, Public Policy and Demographic Change in Overlapping Generations Models

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Contents

Chapter 1

Introduction

In contrast to its Keynesian counterpart, neoclassical macroeconomics prides itself that it is rigorously derived from solid microeconomic foundations. Indeed, the canonical neoclassical macroeconomic model is typically based on the aggregate behaviour of infinitely-lived rational agents maximizing their life-time utility. But, really, how micro-founded are these models? Is it proper to suppose that the aggregate economy acts as though it were one agent? Is it proper to assume that individuals live forever? The commonplace reaction to these questions is, of course, to ignore them under the Friedman norm that if the model is able to replicate reality then it must be fine.

The neoclassical model, however, is not able to replicate reality. This simple observation induced a long line of research trying to incorporate features into large macroeconomic models that would bring them closer to reality. To no avail it seems, for Sims (1980) went so far as to argue that macroeconomics is so out of touch with reality that a straightforward "measurement without theory" approach seemed to outperform the most sophisticated models. Measurement without theory, however, also implies outcomes without policy implications. For the mechanisms at play remain hidden from view.

In a seminal contribution Blanchard (1985) introduced the most basic of human features into an otherwise standard macroeconomic model and came to a surprising conclusion. If non-altruistic individuals are finitely lived, then one of the key theorems of neoclassical thought – the Ricardian equivalence theorem – no longer holds. Innovative as it was, the Blanchard model still suffers from serious shortcomings. For instance, it assumes that individuals have a mortality rate that is independent of their age, that is, a 10-year old child and a 969-year old Methuselah have the same probability of dying (indeed in Blanchard's model there is not even an upper limit for the age of individuals). Furthermore, it assumes that perfect life-insurance markets exist so that, from the point of view of the individual, mortality hardly matters much at all.

In reaction to Blanchard's analysis, a vast body of literature evolved introducing additional features aimed at improving the description of the life-cycle behaviour of the individual who stands at the core of the model. As computing power became more readily available, the so-called computable general equilibrium (CGE) approach was close to follow.

The outward shift in the computational technology frontier made ever more complex models feasible but Sims' (1980) critique seemed to have had a short echo for within foreseeable time these models had again become so complex that the mechanisms translating microeconomic behaviour into macroeconomic outcomes were lost in aggregation and details of the solution algorithm.

The challenge thus remains to construct macroeconomic models that, on the one hand, are solidly founded in the microeconomic environment of the individual agent and, on the other hand, are able to show to the analyst which main mechanisms are at play.[*] In this thesis we contribute our part to this challenge. That is, we construct a series of tractable macroeconomic models that can replicate basic facts of the individual life-cycle and, at the same time, clearly show which mechanisms drive the two-way interaction between microeconomic behaviour and macroeconomic outcomes.

The advantage of this approach over the basic Blanchard (1985) framework is that it can replicate the most important life-cycle choices that an individual makes. The advantage of the approach over the CGE framework is that it retains the flexibility necessary to analyze which factors are driving the relationship between individuals and their macroeconomic environment. Although CGE models can account for numerous institutional traits that are beyond our model, such models fare worse at identifying which mechanisms are at play.

This thesis consists of three substantially independent parts. In the first part we take the assumption of perfect life-insurance markets by the horns and develop a model in which we study the consequences of imperfect annuity markets and use the model to study the effects of different types of taxation and to study how the pension system moderates the impact of a demographic shock. In the second part we return to the theme of annuity markets but ask whether annuities are desirable in the first place. In the final part we focus once more on the impact of demographic changes by studying the different impact of changes in the population growth rate driven by either a change in the birth rate, a change in the mortality rate or a combination of the two.

In a seminal contribution, Yaari (1965) showed that, faced with longevity risk, individuals derive substantial benefits from life-annuities. In fact, in the absence of a bequest motive individuals should invest all their assets in such annuities. Annuities

[*] For an eloquent yet vociferous eulogy of the use of small models in macroeconomics see Turnovsky (2011).

are life-insured financial assets that pay out conditional on the survival of the individual. If the individual survives he receives a premium over and above the market interest. In return, if he dies his savings flow to the annuity firm. Although welfare enhancing from an individual perspective annuity markets are notoriously thin. That is, the availability of annuities is generally limited and, if they exists, annuities are over-priced.

The objective of the chapters 2-4 is to develop an overlapping generations model of a closed economy that incorporates possible imperfections on the annuity market. We introduce these imperfections by allowing for a load factor on the price of annuities that assures that annuities become overpriced. In the second chapter we use the basic framework to study the impact of overpriced annuities on labour-supply and savings decisions at the individual level and growth at the aggregate level. By departing from the classical Blanchard (1985) model and adding sequentially more realistic features we are able to highlight the mechanisms along which the individual life-cycle impedes on the macroeconomic environment. This procedure allows us to stress the importance of both a realistic demographic structure and a realistic labour productivity schedule over the life-cycle. We find that annuity market imperfections have a mild impact on both the aggregate economy and individual decisions.

In the third chapter we apply the basic framework to study the impact of consumption and labour-income taxation. As before, we focus on savings and labour-supply decisions at the individual level and economic growth at the aggregate level. We provide special attention to the way in which the tax income of the government is distributed over the agents. That is, we compare and contrast regimes in which government income is distributed equally over all agents, distributed with a skew toward the elderly or distributed with a skew toward the young. We find that, in principle, the consumption tax redistributes funds from the elderly, who are strong consumers and thus pay the lion's share of tax, to the young, who barely consume but save a lot. The labour income tax, on the other hand, redistributes funds between the working and the idle. Idleness being an attribute of the retired, the tax induces redistribution from saving workers to consuming retirees. Hence, both in welfare and growth terms a consumption tax dominates a labour income tax.

In the fifth chapter we use the model to study the moderating role of the pension system during an ageing shock. We introduce a pay-as-you-go pension system that can be financed on either a defined benefit or a defined contribution basis. In addition, the retirement age may be used as policy parameter. In the wake of an ageing shock we find that the growth rate increases. The rise in the growth rate occurs because individuals have to save for a longer retirement period and, hence, accumulate more capital. This effect is mitigated, though not nullified, if a defined benefit system is in place because the contribution rate has to adjust to accommodate the change in the dependency ratio. Surprisingly, we find that increasing the retirement age dampens economic growth. Intuitively, a higher retirement age decreases the period spent in retirement and therefore the funds necessary to finance it.

Whilst in chapters 2-4 we put emphasis on the consequences of imperfect annuity markets, we do not answer the question of whether these markets are beneficial in the first place. From a microeconomic perspective, we know that annuities are welfare maximizing because they allow for risk sharing between lucky (long-lived) and unlucky (short-lived) individuals. From a macroeconomic perspective, matters are less clear because the microeconomic analysis ignores two key mechanisms. First, the change in savings induced by the opening of an annuities market influences the capital stock and, thereby, wages and the interest rate. Second, in the absence of an annuity market individuals leave accidental bequests that are in one way or the other distributed over the surviving agents.

Thus, the objective of the fifth chapter is to study the general equilibrium effects of opening up an annuity market. Our point of departure is the two period Diamond (1965)-Samuelson (1958) model of overlapping generations. This model contrasts the model in chapters 2-4 in that it does not allow for a very detailed description of the individual life-cycle. However, being more stylized, the model allows us to obtain a full analytical description of the impact, transition and long-run effects of the introduction of annuities. The model features individual agents that can live for a maximum of two periods but transition between the periods is probabilistic. In the absence of annuity markets accidental bequests flow to the government, which can then decide between distributing the funds to the currently young, the currently old or to outright waste them.

Starting from any of these three redistribution possibilities we study the opening up of an annuity market. In line with the microeconomic literature we find that opening up an annuity market is beneficial from an individual perspective. From a macroeconomic perspective, however, matters differ dramatically. We find that there exists a tragedy of annuitization; although full annuitization of assets is privately optimal it may not be socially optimal due to adverse general equilibrium repercussions.

We show that there are two instances of this tragedy. The strong version describes the situation in which accidental bequests were initially wasted by the government. In that case opening up an annuity market induces individuals to save less (because they are now receiving a higher rate on their savings). Because it is welfare enhancing to annuitize all individuals will save less so that the aggregate capital stock and, thereby, wages decline over time. The decline in wages makes individuals worse off, so much that the level of welfare after the introduction of annuities is far less than it was without the annuities.

In the weaker version of the tragedy the government was initially distributing the funds to the young. In that case the introduction of an annuity market sets the economy on a lower welfare path because young agents lose the accidental bequests that they were initially receiving. Part of these were used for savings, hence, their abolishment decreases aggregate capital accumulation because all agents now have less assets to save. Naturally, if the bequests were initially given to the elderly the introduction of an annuity market is welfare enhancing because they eliminate transfers received late in life. These transfers initially acted as a disincentive to save, so that their abolishment increases private, and aggregate, savings.

In the final chapter we return to the analysis of demographic change. However, rather than studying the moderating role of the pension system we use this chapter to study how different types of demographic change affect the aggregate economy. How does a change in the birth rate affect the capital stock? How does a change in the mortality rate affect the capital stock? And what is the impact of a combined mortality and birth rate shock? To analyse these issues, we construct a continuous-time overlapping generations model similar to the one used in the first part of this thesis. However, in contrast to the earlier chapters, in this chapter we focus on an exogenous growth model featuring perfect annuity markets. This set-up allows us to study how demographic changes affect the aggregate capital stock and gives us a basic idea of the dynamics governing the model.

In the theoretical part of the chapter we highlight the mechanisms whereby the demographic life-cycle of the individual agents impedes on the macroeconomic equilibrium. This happens through the "generational turnover term", which refers to the reduction in aggregate consumption due to the addition of newborn agents having no accumulated assets, together with the departure of agents with accumulated lifetime assets. By explicitly setting out the underlying dynamic system, we are able to establish that there are in fact two steady-state equilibria instead of just one as implied in the literature.

The two equilibria contrast sharply in how they are influenced by the demographic structure. In the first equilibrium (the one previously identified in the literature) demographic factors play an important role. They impede on equilibrium per capita consumption directly, through the impact of the mortality function on the discounting of future consumption. In contrast, in the second equilibrium we identify, demographic factors play no direct role, except insofar as they influence the overall population growth rate. The key feature of this equilibrium is that the equilibrium growth rate of consumption just equals the growth rate of population. However, through deeper analysis of this equilibrium we are able to establish that it implies a bubble on the goods market and can only be sustained in the presence of intergenerational or international transfers. Hence, in the remainder of the chapter we focus on the equilibrium previously established in the literature.

To obtain a better understanding of the dynamics governing the model and to prepare for the numerical analysis, we add more demographic structure by focusing on a parameterized mortality function. Using this function we provide an explicit representation of the aggregate macroeconomic dynamic system. This turns out to be a highly nonlinear fifth order system involving not only capital and consumption, as in the standard representative agent economy, but also the dynamics of the various elements of the intergenerational turnover term. This extensive model embeds the classical Blanchard (1985) model, the dynamics of which simplifies dramatically due to the constant mortality assumption.

In our numerical simulations we study the long-run behavior of the model in response to both structural and demographic changes, illustrating their effects on aggregate quantities, as well as on the distributions of consumption and wealth across cohorts. Our numerical results show how the effects of a given increase in the population growth rate contrast sharply – both qualitatively and quantitatively – depending upon whether it occurs through an increase in the birth rate or a decrease in mortality. Whereas in the former case an increase in the population growth rate is associated with a mild decline in the capital stock, in the latter case it leads to a substantial increase in the per capita stock of capital. In addition, a combination between the two can exist such that the impact on the capital stock is exactly off-set. Hence, an increase in the population growth rate can increase, decrease or not affect the capital stock.

As it stands, the final chapter studies mainly theoretical and quantitative issues pertaining to fertility and mortality in the neoclassical framework. However, just as the model in the second chapter served as a stepping stone to the analysis of taxation and pensions in follow-up chapters, this chapter will serve as a stepping stone for the analysis of public policy issues in future research.

Chapter 2

Annuity market imperfections, retirement and economic growth*

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* This chapter is based on Heijdra and Mierau (2009).

2.1 Introduction

One of the most robust findings in economic theory is that individuals facing an uncertain date of death derive great benefits from annuitization. In a seminal paper, Yaari (1965) showed that in the absence of a bequest motive individuals should fully annuitize all of their savings. One of the key assumptions adopted by Yaari concerns the availability of actuarially fair annuities. In a recent paper, Davidoff *et al.* (2005) have demonstrated that the full annuitization result holds in a much more general setting than the one adopted by Yaari, e.g. it obtains also when annuities are less than actuarially fair.

The objective of this chapter is to develop an overlapping generations model of finitely-lived households featuring annuity market imperfections. While the model lays the ground work for the chapters to come, we also use it to study the macroeconomic effects of annuity market imperfections. Are the optimal retirement age and the macroeconomic growth rate significantly affected by the degree of actuarial fairness of annuities or is this imperfection quantitatively unimportant? To answer this question we construct a stylized overlapping generations model of a closed economy featuring endogenous growth due to an inter-firm external effect of the “AK”-type.

Our starting point is the celebrated Blanchard (1985) model, featuring perfect annuities and age-independent mortality (perpetual youth). We extend this model in four directions. First, we endogenize the agent’s life-cycle labour supply decision. Second, we introduce an annuity imperfection parameter, which allows us to study the cases of actuarially fair and unfair annuities in one single framework. In the latter case, annuity firms make profits which are taxed away by the government and redistributed to households. Third, we introduce age-dependent labour efficiency. Fourth, we incorporate the insights of Heijdra and Romp (2008) and postulate an age-dependent mortality process.

Our main findings are as follows. First, the imperfection on the annuity market leads individuals to discount future consumption by their mortality rate as well as their pure rate of time preference. In a perpetual-youth model this leads to a flatter consumption profile. In an age-dependent mortality context this leads to a hump-shaped consumption profile. In both cases capital accumulation, and thereby economic growth, is depressed.

Second, in terms of labour supply we find that both in the perpetual-youth model and the age-dependent productivity model individuals supply less labour during their working life. However, in the perpetual-youth model individuals retire earlier whereas in the age-dependent mortality model individuals retire later. This discrepancy arises because the magnitude of profits made by annuity firms is lower in the age-dependent mortality case. Less profit means lower transfers and thus a smaller wealth effect via that channel.

Third, we show that the way in which annuity firms' profits are recycled plays a key role in the analysis. If these profits are redistributed in the form of lump-sum transfers to households, then the growth and retirement effects of even fairly substantial annuity market imperfections are quantitatively rather small. In contrast, if these profits are drained from the economy via wasteful government consumption, then growth deteriorates dramatically and the retirement age is reduced substantially.

Fourth, our analysis highlights the importance of a correctly modelled demography. Under the Blanchard (1985) assumption the impact of imperfect annuities is grossly overestimated. This is because for a properly modelled demography only the elderly are significantly affected by annuity market imperfections and of these elderly only a small portion is alive at any point in time. Hence, the individual as well as the aggregate effect is mild.

The two papers most closely associated with ours are Bütler (2001) and Hansen and İmrohoroğlu (2008). We extend the insights of Bütler (2001) to the general equilibrium case and explicitly take into account the impact of imperfect annuities on the retirement decision (as opposed to Hansen and İmrohoroğlu (2008)). Furthermore, we also study imperfect annuities in general equilibrium, not only on the individual level as Bütler (2001) and Hansen and İmrohoroğlu (2008).

Like Bütler (2001) and Hansen and İmrohoroğlu (2008) we find that imperfections on the annuity market lead to a hump-shaped profile in consumption. In addition we find that the imperfect annuity market leads to late retirement and depresses economic growth due to less capital accumulation. Furthermore, we find that labour supply during work life decreases. Finally, we show that the profits made by annuity firms play a key role in the analysis.

The remainder of the chapter is structured as follows. Section 2 sets out the core model, whilst section 3 studies the relationship between the annuity market imperfection, the retirement decision, and macroeconomic growth. Section 4 introduces the two extensions that allow our model to better resemble realistic features of life-cycle choices. Section 5 concludes.

2.2 Model

2.2.1 Firms

The production side of the model makes use of the insights of Romer (1989) and postulates the existence of sufficiently strong external effects operating between private firms in the economy. There is a large and fixed number, $\mathcal{N}$, of identical, perfectly competitive firms. The technology available to firm i is given by:

$$Y_i(t) = \Omega(t) K_i(t)^{\varepsilon_K} L_i(t)^{1-\varepsilon_K}, \qquad 0 < \varepsilon_K < 1, \tag{2.1}$$

where $Y_i(t)$ is output, $K_i(t)$ is capital use, $L_i(t)$ is the labour input, and $\Omega(t)$ represents the general level of factor productivity which is taken as given by individual firms. The competitive firm hires factors of production according to the following marginal productivity conditions:

$$\begin{aligned} w(t) &= (1-\varepsilon_K)\,\Omega(t)\,\kappa_i(t)^{\varepsilon_K}, &(2.2)\\ r(t)+\delta &= \varepsilon_K \Omega(t)\,\kappa_i(t)^{\varepsilon_K-1}, &(2.3) \end{aligned}$$

where $\kappa_i(t) \equiv K_i(t)/L_i(t)$ is the capital intensity, $w(t)$ is the wage rate, $r(t)$ is interest rate and δ is the depreciation rate. The rental rate on each factor is the same for all firms, i.e. they all choose the same capital intensity and $\kappa_i(t) = \kappa(t)$ for all $i = 1, \cdots, \mathcal{N}$. This is a very useful property of the model because it enables us to aggregate the microeconomic relations to the macroeconomic level.

Generalizing the insights of Saint-Paul (1992, p. 1247) and Romer (1989) to a growing population, we assume that the inter-firm externality takes the following form:

$$\Omega(t) = \Omega_0 \kappa(t)^{1-\varepsilon_K}, \tag{2.4}$$

where Ω_0 is a positive constant, $\kappa(t) \equiv K(t)/L(t)$ is the economy-wide capital intensity, $K(t) \equiv \sum_i K_i(t)$ is the aggregate capital stock, and $L(t) \equiv \sum_i L_i(t)$ is aggregate employment. According to (2.4), total factor productivity depends positively on the aggregate capital intensity, i.e. if an individual firm i raises its capital intensity, then *all* firms in the economy benefit somewhat because the general productivity indicator rises for all of them. Using (2.4), equations (2.1)–(2.3) can now be rewritten in aggregate terms:[1]

$$\begin{aligned} Y(t) &= \Omega_0 K(t), && (2.5)\\ w(t)L(t) &= (1-\varepsilon_K)Y(t), && (2.6)\\ r(t) &= r = \varepsilon_K \Omega_0 - \delta, && (2.7) \end{aligned}$$

where $Y(t) \equiv \sum_i Y_i(t)$ is aggregate output and we assume that capital is sufficiently productive, i.e. $\varepsilon_K \Omega_0 - \delta > 0$. The aggregate technology is linear in the capital stock and the interest rate is constant.[2]

2.2.2 Consumers

Individual behaviour

We generalize the Blanchard (1985) model of consumer behaviour by including an endogenous labour-leisure decision and by assuming potentially imperfect annuity markets. At time t, expected remaining-lifetime utility of an individual born at time v

[1] All firms use the same capital intensity ($\kappa_i(t) = \kappa(t)$), so that $Y_i(t) = \Omega(t) L_i(t) \kappa(t)^{\varepsilon_K}$ and $Y(t) = L(t)\Omega(t)\kappa(t)^{\varepsilon_K}$. By using (2.4) in this expression, we find (2.5). For the wage we find $w(t) = (1-\varepsilon_K)\Omega(t)\kappa(t)^{\varepsilon_K} = (1-\varepsilon_K)\Omega_0\kappa(t)$, which can be rewritten to get (2.6). Finally, for the rental rate on capital we find $r(t) + \delta = \varepsilon_K \Omega(t) \kappa(t)^{\varepsilon_K - 1} = \varepsilon_K \Omega_0$.

[2] Romer (1989, p. 90) makes $\Omega(t)$ dependent on the stock of capital $K(t)$, an approach also adopted by Saint-Paul (1992, p. 1247). Romer rationalizes his formulation by appealing to the public good character of knowledge and by assuming that physical capital and knowledge are produced in constant proportions. Both Romer and Saint-Paul assume a constant labour force. In order to accommodate population growth, we make the knowledge spillover dependent on the capital intensity. Note that the original Romer specification would result in $r(t) + \delta = \varepsilon_K \Omega_0 L(t)^{\varepsilon_K - 1}$, i.e. a downward trend in the real interest rate *contra* Kaldor's stylized facts.

$(v \leq t)$ is given by:

$$\mathbb{E}\Lambda\left(v,t\right) \equiv \int_t^{\infty} \ln\left[C(v,\tau)^{\varepsilon_C} \cdot \left[1 - L(v,\tau)\right]^{(1-\varepsilon_C)}\right] \cdot e^{(\rho+\mu)(t-\tau)} d\tau, \tag{2.8}$$

where $C\left(v,\tau\right)$ is consumption, $L\left(v,\tau\right)$ is labour supply (the time endowment is equal to unity), ρ is the pure rate of time preference, and μ is the instantaneous mortality rate.[3]

The agent's budget identity is given by:

$$\dot{A}(v,\tau) = r^A A(v,\tau) + w(\tau)L\left(v,\tau\right) - C(v,\tau) + TR\left(v,\tau\right), \tag{2.9}$$

where $A\left(v,\tau\right)$ is the stock of financial assets, r^A is the annuity rate of interest, $w\left(\tau\right)$ is the wage rate, and $TR\left(v,\tau\right)$ are lump-sum transfers from the government (see below), all defined in real terms. Following Yaari (1965), we postulate the existence of annuity markets, but unlike Yaari we allow the annuities to be less than actuarially fair. Since the agent is subject to lifetime uncertainty and has no bequest motive, he/she will fully annuitize so that the annuity rate of interest facing the agent is given by:

$$r^A \equiv r + \theta\mu, \tag{2.10}$$

where r is the real interest rate (see (2.7)), and θ is a parameter $(0 \leq \theta \leq 1)$. Our specification for the annuity rate can be rationalized in three ways. First, $1-\theta$ may be interpreted as a load factor needed to cover the administrative costs of organizing the annuity firm – see Horneff *et al.* (2008, p. 3595). Second, as Hansen and İmrohoroğlu (2008, p. 569) suggest, θ may represent the fraction of assets that are annuitized. Provided θ is strictly less that unity, there will be unintended bequests under this interpre-

[3] As is well known from the Real Business Cycle (RBC) literature, in the presence of technological change, certain restrictions must be imposed on preferences in order to allow for a meaningful steady state to exist. King *et al.* (2002, p. 94-95) show that the only admissible felicity functions take the following form:

$$U \equiv \frac{1}{1-\sigma} C^{1-\sigma} \cdot v\left(1-L\right), \qquad \sigma \neq 1, \quad \sigma > 0 \tag{A}$$

$$\equiv \ln C + v\left(1-L\right) \tag{B}$$

With specification (A), $v\left(1-L\right)$ must be increasing and concave if $\sigma < 1$ and decreasing and convex if $\sigma > 1$. Further restrictions are needed to ensure overall concavity. Under specification (B), all we need is that $v\left(1-L\right)$ is increasing and concave. Our function is a special case of (B) with $v\left(1-L\right)$ log-linear in leisure.

tation. Third, annuity firms may possess some market power, allowing them to make a profit by offering a less than actuarially fair annuity rate. In this chapter, we adopt the market-power interpretation. We shall refer to $1-\theta$ as the degree of imperfection in the annuity market.[4]

Our specification is quite general and incorporates three important cases:

- *Perfect annuities* (PA). The case of perfect (actuarially fair) annuities is obtained by setting $\theta = 1$. Life insurance companies break even, and $TR\,(v, \tau) = 0$.

- *Imperfect annuities* (IA). The case of imperfect (less than actuarially fair) annuities is obtained by assuming $0 < \theta < 1$. Life insurance companies make excess profits, $\mu\,(1-\theta)\,A\,(\tau)$, which are taxed away by the government and distributed in a lump-sum fashion to surviving agents.

- *No annuities* (NA). For $\theta = 0$ there are no annuity markets. The agent can save at the interest rate r, but borrowing is impossible because, with lifetime uncertainty, he/she faces a probabilistic time-of-death wealth constraint of the form, $\text{prob}\,\{A\,(v,\tau) \geq 0\} = 1$ (Yaari, 1965, p. 139). By definition, $TR\,(v, \tau) = 0$.

In the remainder of this chapter we restrict attention to the PA and IA cases.

The agent chooses time profiles for $C\,(v,\tau)$, $A\,(v,\tau)$, and $L\,(v,\tau)$ (for $\tau \geq t$) in order to maximize (2.1), subject to (i) the budget identity (2.2), (ii) a No Ponzi Game (NPG) condition, $\lim_{\tau\to\infty} A\,(v,\tau)\,e^{(r+\theta\mu)(t-\tau)} = 0$, (iii) the initial asset position in the planning period, $A\,(v,\tau)$, and (iv) a non-negativity condition, $L\,(v,\tau) \geq 0$. We restrict attention to the optimal individual life-cycle decisions in the context of an economy moving along a steady-state balanced growth path.

Along the balanced growth path, labour productivity grows at a constant exponential rate, γ (see below), and as a result individual agents face an upward sloping path

[4] Another explanation for the overpricing of annuities is adverse selection (Finkelstein and Poterba, 2002). That is, agents with a low mortality rate are more likely to buy annuities than agents with high mortality rates. However, because mortality is private information annuity firms "mis-price" annuities for low-mortality agents, thus creating a load factor. Abel (1986) and Heijdra and Reijnders (2009) study this adverse selection mechanism in a general equilibrium model featuring healthy and unhealthy people and with health status constituting private information. The unhealthy get a less than actuarially fair annuity rate whilst the healthy get a better than actuarially fair rate for part of life. An alternative source of imperfection may arise from the way that the annuity market is structured. Yaari (1965) assumes that there is a continuous spot market for annuities. In reality, however, investments in annuities are much lumpier. See Pissarides (1980) for an early analysis of this issue.

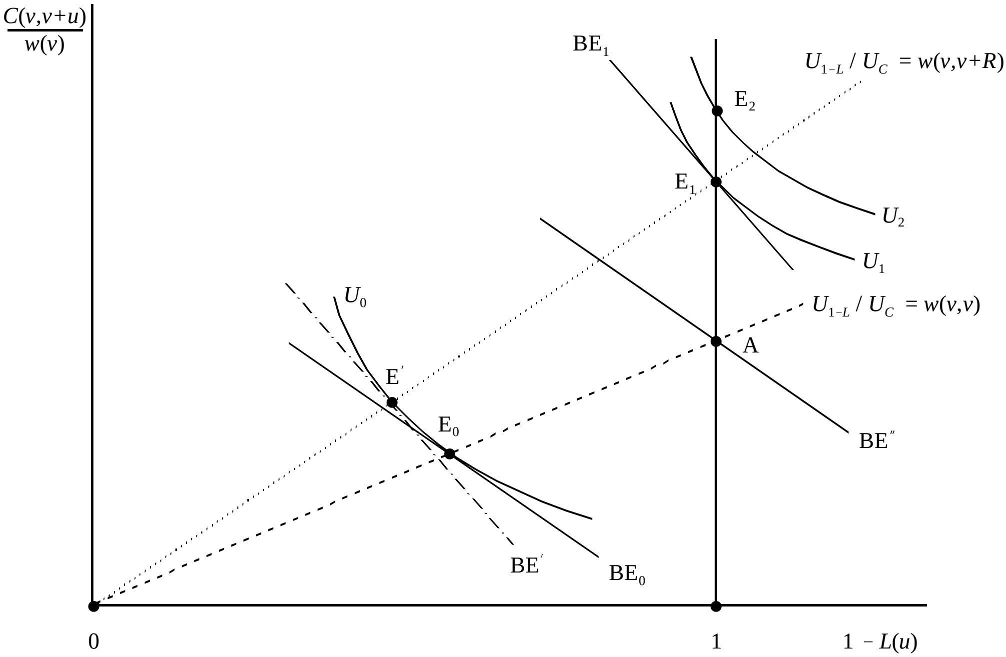

Figure 2.1. Life-cycle consumption, labour supply, and retirement

for real wages over their lifetimes:

$$w(\tau) = w(v)\, e^{\gamma(\tau - v)}. \tag{2.11}$$

The individual consumption Euler equation is given by:

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = r - \rho - (1-\theta)\,\mu > 0. \tag{2.12}$$

With imperfect annuities, individual consumption growth is affected by the mortality rate, a result first demonstrated for the case with $\theta = 0$ by Yaari (1965, p. 143). During the working period, the agent equates the marginal rate of substitution between leisure and consumption to the wage rate at all times:

$$\frac{(1-\varepsilon_C)\,/\,(1-L(v,\tau))}{\varepsilon_C / C(v,\tau)} = w(\tau). \tag{2.13}$$

The consumption-leisure choice is illustrated in Figure 1, where $C(v, v+u)\,/w(v)$ and

$L(u)$ stand for, respectively, consumption (scaled by the wage rate at birth) and labour supply of the agent at age u. The initial choice at age $u = 0$ is at point E_0 where there is a tangency between an indifference curve (labeled U_0) and a "budget line" (labeled BE_0).[5] If there were no economic growth, the wage rate would be constant over the agent's lifetime and the optimum would gradually move along the dashed line from E_0 to A at which point it is optimal to retire. This move reflects the positive wealth effect on the demands for consumption and leisure. After retirement, the agent would move along the vertical leisure constraint in the direction of points E_1 and E_2.

Matters are slightly more complicated in the presence of economic growth and an upward sloping wage profile (2.11). Over the agent's life the utility-expansion path rotates in a counter-clockwise fashion inducing substitution effects. In terms of Figure 1, the agent retires at point E_1 where the marginal rate of substitution between leisure and consumption is equal to $w(R)$, where R stands for this agent's *age* at retirement. Using the dotted utility-expansion line through point E_1 we find that the total effect on consumption and leisure during working life is given by the move from E_0 to E_1. The pure substitution effect is given by the move from E_0 to E', and the wealth effect is the move from E' to E_1.

Armed with this graphical apparatus we can explain the following analytical expressions. Consumption of a newborn is given by:

$$C(v,v) = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C + (1-\varepsilon_C)\left[1-e^{-(\rho+\mu)R(v)}\right]} \cdot LI(v,v), \tag{2.14}$$

where $R(v)$ is the retirement age chosen by an agent born at time v, and $LI(v,v)$ is lifetime income of the agent:

$$LI(v,v) = w(v) \cdot \frac{1-e^{-(r-\gamma+\theta\mu)R(v)}}{r-\gamma+\theta\mu} + LT(v,v), \tag{2.15}$$

[5] During the working period, the budget line is given by:

$$X(v,\tau) = w(\tau)\left[1-L(v,\tau)\right] + C(v,\tau),$$

where $X(v,\tau)$ is full consumption. The line BE_0 is obtained by substituting $X(v,v)$.

where $LT(v,v)$ are lifetime transfers received from the government:

$$LT(v,v) \equiv \int_v^\infty TR(v,\tau)\, e^{(r+\theta\mu)(v-\tau)} d\tau. \tag{2.16}$$

Equation (2.14) shows that consumption of a newborn is proportional to lifetime income. The marginal propensity to consume out of lifetime income is decreasing in the retirement age. Equation (2.15) provides the definition of lifetime income. The first term on the right-hand side represents the present value of the time endowment during working life, using the growth-corrected annuity rate of interest $(r-\gamma+\theta\mu)$ for discounting. The later one retires, the larger is this term. The second term on the right-hand side of (2.15) is just the present value of transfers, defined in (2.16).

Point E_1 in Figure 1 is attained at the point where consumption satisfies:

$$C(v, v+R(v)) = \frac{\varepsilon_C}{1-\varepsilon_C} w(v)\, e^{\gamma R(v)}. \tag{2.17}$$

By using (2.12) we find that $C(v,\tau) = C(v,v)\, e^{(r-\rho-(1-\theta)\mu)(\tau-v)}$ so that (2.17) can be rewritten as:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-[r-\gamma-\rho-(1-\theta)\mu]R(v)}. \tag{2.18}$$

Equations (2.14) (with (2.15) substituted), and (2.18) represent a simultaneous system implicitly determining $C(v,v)/w(v)$ and $R(v)$ as a function of the structural parameters (ε_C, ρ, μ, r, and θ), the macroeconomic growth rate (γ), and scaled lifetime transfers ($LT(v,v)/w(v)$).

We illustrate the optimal retirement choice in Figure 2. This figure is based on the following parameter settings. The interest rate is set at six percent per annum ($r = 0.06$) whilst the rate of time preference is three and a half percent ($\rho = 0.035$). These values imply that in the presence of perfect annuities, individual consumption grows at 2.5 percent per annum (see (2.12)). The instantaneous mortality rate is estimated with Dutch mortality data for the cohort born in 1960 (see below for details). This yields a value of 1.26 percent per annum ($\mu = 0.0126$), implying an expected remaining lifetime of 79.4 years. We assume that labour productivity growth equals two percent per annum ($\gamma = 0.02$), and set the utility parameter for consumption at such a value that the optimal retirement age with perfect annuities is $R = 42$ years. This yields a

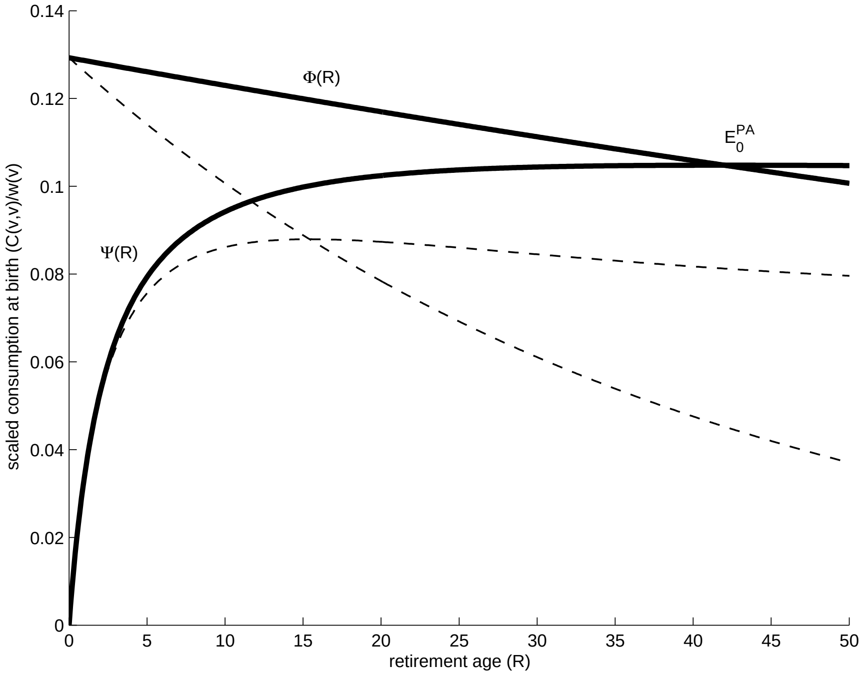

Figure 2.2. Optimal retirement age

value of $\varepsilon_C = 0.1145$. Finally, we assume that annuities are perfect, i.e. $\theta = 1$ in Figure 2. This simplifies matters somewhat because $LT\,(v, v) = 0$ for this case.

In Figure 2, the $\Psi\,(R)$ function plots the combinations between $C\,(v, v)\,/w\,(v)$ and $R\,(v)$ implied by equations (2.14)–(2.15) (with $LT\,(v, v) = 0$ imposed). Despite the fact that the marginal propensity to consume is a downward sloping function of the retirement age, lifetime income is sharply increasing in the retirement age and $\Psi\,(R)$ is upward sloping as a result. The downward sloping $\Phi\,(R)$ function plots equation (2.18) and intersects $\Psi\,(R)$ at point E_0^{PA}. There is a unique optimal retirement age which, for the parameters used here, equals $R = 42$.

Figure 2 also illustrates the partial equilibrium effects of a change in the macroeconomic growth rate, γ. Indeed, the thin dashed lines depict the $\Phi\,(R)$ and $\Psi\,(R)$ functions for the zero-growth case ($\gamma = 0$), for which the optimal retirement age is $R = 15.6$ years. In terms of Figure 1, this is the case where the agent moves from E_0

to A along the dashed utility-expansion curve. With a flat wage profile, equilibrium consumption at birth (and at all ages) and the retirement age are both lower than with an upward sloping wage profile. Finally, we note that an increase in lifetime transfers leads to an upward shift in $\Psi(R)$, higher consumption at birth and a lower retirement age. The transfers thus cause a negative wealth effect on the optimal retirement age.

With imperfect annuities ($0 < \theta < 1$) we must confront the issue of redistribution of excess profits and recognize the fact that $LT(v,v)$ will be positive in general. To keep things as simple as possible, we assume that the lump-sum transfers are set according to:

$$TR(v,\tau) = z \cdot w(\tau), \tag{2.19}$$

where z is a positive parameter, that is taken as given by individual agents but determined endogenously in general equilibrium via the balanced budget requirement of the redistribution scheme (see below). By using (2.19) in (2.16) we find:

$$\frac{LT(v,v)}{w(v)} \equiv \frac{z}{r - \gamma + \theta\mu}. \tag{2.20}$$

It is easy to show that z is constant along the balanced growth path.[6] Equations (2.14)–(2.15), (2.18), and (2.20) in combination imply that the retirement age is independent of v, i.e. $R(v) = R$ for all v. We summarize this important result in the following proposition.

Proposition 2.1. *Consider lump-sum redistribution of excess profits of life-insurance companies, of the form $TR(v,\tau) = z \cdot w(\tau)$. In that case: (i) the optimal retirement age is independent of v, i.e. $R(v) = R$ for all v; (ii) the optimal ratio between consumption at birth and the wage rate at birth is independent of v, i.e. $C(v,v)/w(v) = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho-(1-\theta)\mu)R}$ for all v.*

[6] An alternative feasible redistribution scheme would set $TR(v,\tau) = z \cdot w(v)$, implying that $LT(v,v)/w(v) = z/(r+\theta\mu)$ along the balanced growth path. Interestingly, "actuarially fair" lump-sum redistribution, setting transfers according to $TR(v,\tau) = \mu(1-\theta)A(v,\tau)$, is infeasible. Under such a scheme, $LT(v,v)$ becomes unbounded which is clearly infeasible.

Aggregate household behaviour

In this subsection we derive expressions for per-capita average consumption, saving, and labour supply. We allow for constant population growth π and distinguish between the birth rate, β, and the mortality rate rate, μ, so that $\pi \equiv \beta - \mu$. The relative cohort weights evolve according to:

$$p(v,t) \equiv \frac{P(v,t)}{P(t)} = \beta e^{\beta(v-t)}, \qquad t \geq v, \tag{2.21}$$

where $P(v,t)$ is the size of cohort v at time t and $P(t)$ is the total population. Using (2.21), we can define per-capita average values in general terms as:

$$x(t) \equiv \int_{-\infty}^{t} p(v,t)\, X(v,t)\, dv, \tag{2.22}$$

where $X(v,t)$ denotes the variable in question at the individual level and $x(t)$ is the per capita average value of the same variable.

Off the steady-state growth path, exact analytical aggregation of the individual behavioural decision rules is impossible. To see why this is the case, note, for example, that consumption of workers features an age-dependent propensity to consume out of age-dependent wealth making aggregation impossible. We, therefore, focus on steady-state relationships. We know that $R(v) = R$ for all v, so for consumption we find:

$$\begin{aligned} C(v,v) &= \frac{\varepsilon_C}{1-\varepsilon_C} w(v)\, e^{-[r-\gamma-\rho-(1-\theta)\mu]R}, & (2.23)\\ C(v,t) &= C(v,v)\, e^{[r-\rho-(1-\theta)\mu](t-v)}, & (2.24) \end{aligned}$$

whilst the wage rate satisfies equation (2.11). Using (2.22), per capita average consumption is thus given by:

$$c(t) \equiv \int_{-\infty}^{t} p(v,t)\, C(v,t)\, dv \equiv \frac{C(v,v)}{w(v)} \cdot \frac{\beta w(t)}{\gamma + \beta + \rho + (1-\theta)\mu - r}. \tag{2.25}$$

It follows from (2.13) and (2.23)-(2.24) that labour supply of workers in period t

$(t - v \leq R)$ can be written as:

$$L(v,t) = 1 - e^{-[r-\gamma-\rho-(1-\theta)\mu](R+v-t)}. \tag{2.26}$$

Since $L(v,t) = 0$ for retirees $(t - v > R)$, per capita average labour supply is equal to:

$$\begin{aligned} l(t) &\equiv \int_{t-R}^{t} p(v,t) L(v,t)\, dv \\ &= \left[1 - e^{-\beta R}\right] - \beta e^{-\beta R} \cdot \frac{e^{[\gamma+\beta+\rho+(1-\theta)\mu-r]R} - 1}{\gamma+\beta+\rho+(1-\theta)\mu - r} \equiv l, \end{aligned} \tag{2.27}$$

with $0 < l < 1$. The term in square brackets on the right-hand of (2.27) provides the first mechanism by which l falls short of unity: agents retire and their unit time endowment is consumed in full in the form of leisure. The second composite term on the right-hand side of (2.27) represents the other mechanism by which l falls short of unity: as workers age they reduce their labour supply.

At the individual level, financial assets are accumulated according to:

$$\dot{A}(v,t) = (r + \theta\mu) A(v,t) + w(t) L(v,t) + zw(t) - C(v,t), \tag{2.28}$$

where $L(v,t) = 0$ for retirees (for $t - v > R$). Per capita aggregate assets are defined as $a(t) \equiv \int_{-\infty}^{t} p(v,t) A(v,t)\, dv$ so that:

$$\dot{a}(t) = \int_{-\infty}^{t} p(v,t) \dot{A}(v,t)\, dv - \beta a(t), \tag{2.29}$$

where we have incorporated the fact that individual agents are born bare of financial assets $(A(v,v) = 0)$ and that cohort shares evolve over time according to $\dot{p}(v,t) = -\beta p(v,t)$. Substituting (2.28) into (2.29) and noting (2.27) we obtain:

$$\dot{a}(t) = (r + \theta\mu - \beta) a(t) + w(t) l(t) + zw(t) - c(t). \tag{2.30}$$

The balanced-budget requirement for the lump-sum redistribution scheme is given in per capita terms by:

$$\mu(1-\theta) a(t) = zw(t). \tag{2.31}$$

Table 2.1. Balanced growth and retirement in the core model

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C + (1-\varepsilon_C)\left[1-e^{-(\rho+\mu)R}\right]} \cdot \frac{1-e^{-(r-\gamma+\theta\mu)R}+z}{r-\gamma+\theta\mu} \tag{T1.1}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho-(1-\theta)\mu)R} \tag{T1.2}$$

(b) Macroeconomic relationships:

$$z = \mu(1-\theta)\frac{k(t)}{w(t)} \tag{T1.3}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[l - \frac{c(t)}{w(t)}\right] \cdot \frac{w(t)}{k(t)} \tag{T1.4}$$

$$\frac{w(t)\,l}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \tag{T1.5}$$

$$l \equiv 1 - e^{-\beta R} - \beta e^{-\beta R}\frac{e^{[\gamma+\beta+\rho+(1-\theta)\mu-r]R}-1}{\gamma+\beta+\rho+(1-\theta)\mu-r} \tag{T1.6}$$

$$\frac{c(t)}{w(t)} \equiv \frac{\beta}{\gamma+\beta+\rho+(1-\theta)\mu-r} \cdot \frac{C(v,v)}{w(v)} \tag{T1.7}$$

Definitions: Endogenous are $C(v,v)/w(v)$, R, z, γ, l, $w(t)/k(t)$, and $c(t)/k(t)$. Parameters: birth rate β, mortality rate μ, population growth rate $\pi \equiv \beta - \mu$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K \Omega_0 - \delta$, where δ is the depreciation rate of capital.

Finally, by substituting (2.31) into (2.30) we obtain:

$$\dot{a}(t) = (r+\mu-\beta)\,a(t) + w(t)\,l(t) - c(t). \tag{2.32}$$

Like in the standard case with perfect annuities, the aggregate per capita annuity receipts, $\theta\mu a(t)$, do not feature directly in (2.32) because they constitute pure transfers from the dead to the living. In each period, life insurance companies receive $\mu a(t)$ from the estates of the deceased and pay $\theta\mu a(t)$ to their surviving customers. The resulting profit, $(1-\theta)\,\mu a(t)$, is taxed away by the government and redistributed to the surviving agents. The transfers are eliminated from the per capita average asset accumulation equation.

2.2.3 Balanced growth path

The capital market equilibrium condition is given by $A(t) = K(t)$. In per capita average terms we thus find:

$$a(t) = k(t), \tag{2.33}$$

where $k(t) \equiv K(t)/P(t)$ is the per capita stock of capital. From (2.5)-(2.6) we easily find:

$$\begin{aligned} y(t) &= \Omega_0 k(t), && (2.34) \\ w(t)\,l(t) &= (1-\varepsilon_K)\,y(t), && (2.35) \end{aligned}$$

where $y(t) \equiv Y(t)/P(t)$ is per capita output.

The macroeconomic growth model has been written in a compact format in Table 1. Equation (T1.1) is obtained by substituting (2.15) and (2.20) into (2.14). Equation (T1.2) is the same as (2.23). Equation (T1.3) is (2.31) with (2.33) substituted. Equation (T1.4) is obtained by substituting (2.33) into (2.32). Equation (T1.5) is obtained by combining (2.34)-(2.35) and noting (2.27). Equation (T1.6) is the same as (2.27). Finally, (T1.7) is the same as (2.25).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Equations (T1.1)-(T1.2) determine scaled newborn consumption, $C(v,v)/w(v)$, and the optimal retirement age, R, as a function of the key macroeconomic variables. Equations (T1.3)-(T1.7) determine equilibrium transfers, z, the macroeconomic growth rate, γ, the overall wage-capital ratio, $w(t)/k(t)$, aggregate labour supply, l, and the $c(t)/w(t)$ ratio as a function of the optimal retirement age and scaled newborn consumption.

2.3 Retirement, growth and annuities

In this section we compute and visualize the comparative static general equilibrium effects for the core model of Table 1. To compute the initial general equilibrium we assume that annuities are perfect ($\theta = 1$) and use the coefficient values mentioned above (in the paragraph below equation (2.18)). We assume that rate of population growth is one percent per annum ($\pi = 0.01$). Since $\pi \equiv \beta - \mu$, this implies that, for the mortality rate that was postulated above, the birth rate is $\beta = 0.0226$. The capital depreciation rate is ten percent per annum ($\delta = 0.10$). We use the efficiency parameter of capital as a calibration parameter and find $\varepsilon_K = 0.8348$.[7] It follows that the constant in the production function is equal to $\Omega_0 = (r + \delta) / \varepsilon_K = 0.1917$. The initial steady-state growth path has the following features: $C(v, v) / w(v) = 0.1048$, $R = 42$, $z = 0$, $\gamma = 0.02$, $l = 0.0691$, $c(t) / w(t) = 0.1346$, and $w(t) / k(t) = 0.4583$. For convenience these values are restated in the first column in Table 2(a).

Figure 3 visualizes some of the key features of the calibration. Figure 3(a) depicts the *general equilibrium* determination of the retirement age and the macroeconomic growth rate. The solid line represents the microeconomic equilibrium condition, i.e. it depicts (γ, R) combinations for which (T1.1) and (T1.2) are equated (recall that $z = 0$ in the base case, so the microeconomic equilibrium can be computed conditional on the macroeconomic growth rate only). In Figure 3(a), the dashed line depicts the macroeconomic equilibrium conditions, i.e. it depicts (γ, R) combinations for which (T1.3)–(T1.7) are satisfied. The equilibrium is at point E_0, where the two lines intersect.

Figure 3 also illustrates the steady-state age profiles for the key variables (solid lines). Figure 3(b) shows that scaled consumption is exponential in the agent's age. Figure 3(c) shows that the agent gradually reduces the number of hours supplied to the labour market, and retires permanently at age $R = 42$. Finally, Figure 3(d) shows that the path of financial assets is monotonically increasing in age, and features a slight kink at the retirement age.

Next we consider the equilibrium under imperfect annuities. Instead of setting $\theta = 1$, we simulate the model with a value of $\theta = 0.70$ and keep all other parameters the same.[8] The new equilibrium values for the different variables are reported in the second column in Table 2(a). Obviously, with imperfect annuities lump-sum transfers become positive. Interestingly, agents reduce lifetime labour supply slightly but retire at about the same age as under perfect annuities.

[7] This is, of course, an implausibly high value, signalling that it is hard to obtain a calibration for the core model that yields plausible values for all parameters. Below we introduce some model extensions that allow us to substantially improve the quality of the calibration in this respect.

[8] Friedman and Warshawsky (1988, p. 59) estimate a load factor of 48 cents per dollar of expected present value. They suggest that 15 cents of this amount may be due to adverse selection and the remaining 33 cents due to costs, taxes, and profit.

The new growth rate is about thirty five basis points lower than under perfect annuities. In Figure 4 we visualize the general equilibrium effects of θ on the retirement decision and scaled consumption of a newborn. The solid lines depict the case with perfect annuities ($\theta = 1$). The equilibrium is at point E^{PA}. The thick dashed lines illustrate the case with imperfect annuities ($\theta = 0.70$), taking into account the general equilibrium effects on γ and z. The equilibrium with imperfect annuities is at point E^{IA}, which lies north-east of point E^{PA}. Agents retire slightly later on in life and consume more at birth. The thinly dashed line in Figure 4 depicts the $\Psi\left(R\right)$-line for imperfect annuities, but assuming that the transfers are zero. The total effect of the move from E^{PA} to E^{IA} can thus be decomposed into a part that is caused by the effect of the growth rate, and a part that is caused by lump-sum transfers.

In order to better understand these growth effects, we use (2.34), (T1.5) in (T1.4) to obtain:

$$\gamma = r - \pi + \Omega_0 \cdot \left[1 - \varepsilon_K - \frac{c\left(t\right)}{y\left(t\right)}\right]. \tag{2.36}$$

The model features an inverse relationship between the growth rate and the macroeconomic consumption-output ratio. In the bottom row of Table 2(a) we find that the decrease in the growth rate is accompanied by an increase in the consumption-output ratio from 0.3217 to 0.3398.

The new steady-state age profiles for the imperfect annuity case have been illustrated in Figures 3(b)–(d) (see the dashed lines). The growth rate in individual consumption is reduced somewhat because $-\left(1-\theta\right)\mu$ features in equation (2.12). Figure 3(c) shows that the agent reduces labour supply especially at early age levels. Finally, Figure 3(d) shows that the age profile for scaled financial assets continues to be upward sloping, though it is lower than under perfect annuities.

2.4 Extensions

In the previous section we used a calibrated version of the core model to show that an imperfection in the annuity market leads to a slight increase in the optimal retirement age and a decrease in the macroeconomic growth rate. The core model, though useful for analytical purposes, suffers from a number of empirical deficiencies. These are:

Table 2.2. Growth and retirement: quantitative effects

	(a) *Core case*		(b) *Productivity*		(c) *Mortality*		(d) *Combined*		
	$\theta = 1.0$	$\theta = 0.7$	$\theta = 1.0$	$\theta = 0.7$	$\theta = 1.0$	$\theta = 0.7$	$\theta = 1.0$	$\theta = 0.7$	
								(i)	(ii)
$\dfrac{C(v,v)}{w(v)}$	0.1048	0.1062	0.0926	0.0934	0.1028	0.1057	0.1017	0.1007	0.0963
S (years)	0	0	7.47	7.65	0	0	7.47	7.40	6.75
R (years)	42	42.02	42	41.83	42	48.78	42	42.40	39.97
z	0	0.0078	0	0.0044	0	0.0031	0	0.0025	0
γ (%)	2.00	1.65	2.00	1.64	2.00	1.92	2.00	1.89	1.69
l (or n)	0.0691	0.0651	0.0831	0.0802	0.0825	0.0804	0.1100	0.1078	0.1060
$\dfrac{c(t)}{w(t)}$	0.1346	0.1340	0.1188	0.1189	0.1179	0.1157	0.1166	0.1144	0.1158
$\dfrac{w(t)}{k(t)}$	0.4583	0.4863	0.8401	0.8701	0.8482	0.8712	4.6038	4.6995	4.7792
$\dfrac{c(t)}{y(t)}$	0.3217	0.3398	0.4343	0.4501	0.4348	0.4384	0.8050	0.8067	0.8300

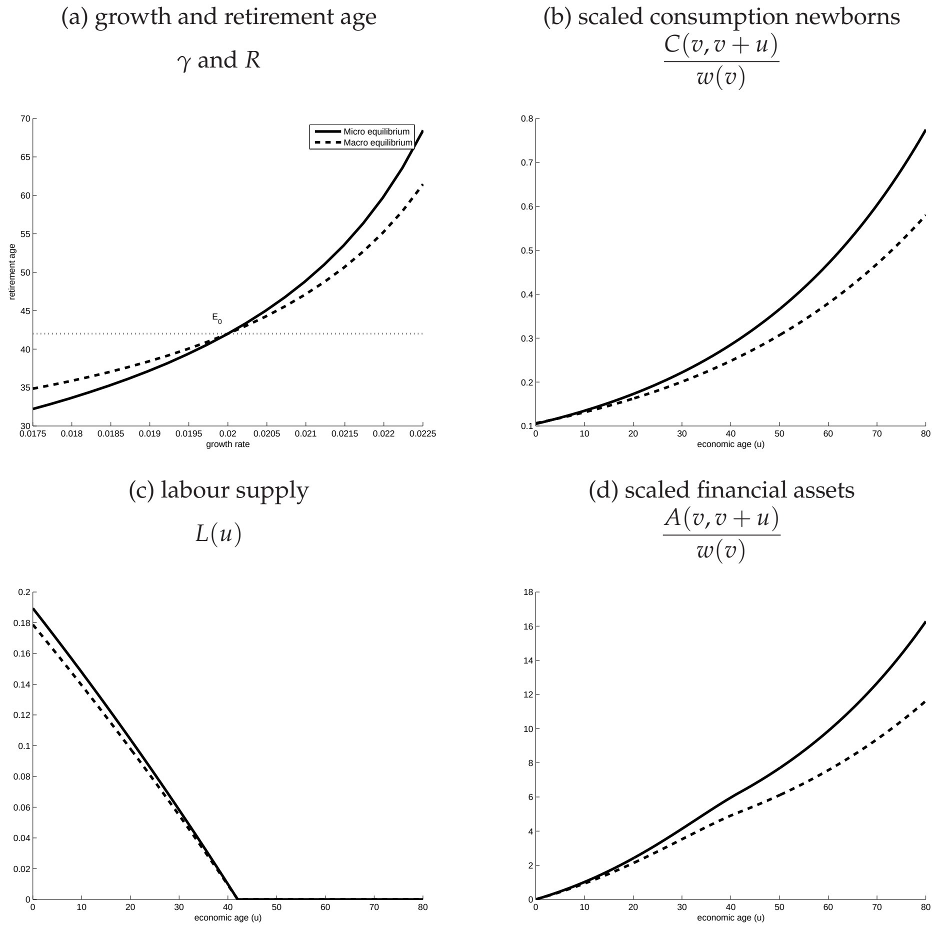

Figure 2.3. General equilibrium in the core model

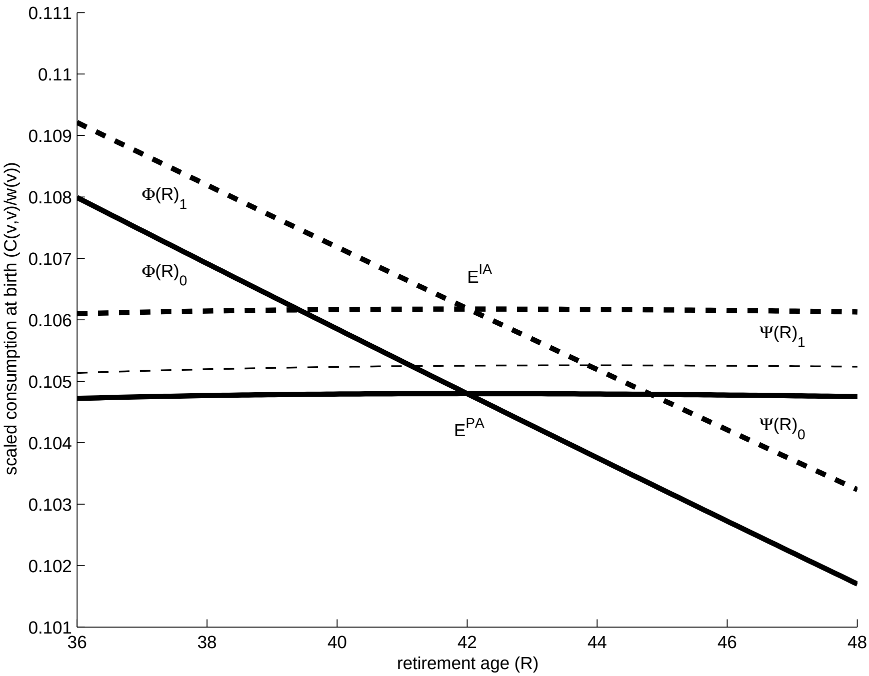

Figure 2.4. Imperfect annuities and the retirement date

(ED1). The age profile for consumption is monotonically increasing, whereas it is hump-shaped in reality (Gourinchas and Parker, 2002, and Fernández-Villaverde and Krueger, 2007).

(ED2). The age profile for labour supply is monotonically decreasing. In reality, labour supply is constant and age-invariant for most of working life and tapers off rapidly near the optimal retirement age (see, for example, McGrattan and Rogerson (2004) for the United States).

(ED3). Labour productivity is age-independent, whereas in reality it appears to be hump-shaped (cf. Hansen, 1993 and Rios-Rull, 1996).

(ED4). Under perfect annuities, the age profile for financial assets is monotonically rising. In reality, financial assets (a) display a hump-shaped profile, and (b) remain non-negative in old age (Huggett, 1996).

(ED5). To calibrate the model for a realistic retirement age and macroeconomic growth rate, an implausibly high efficiency parameter for capital must be postulated.

In this section we consider two important model extensions, namely age-dependent labour productivity and age-dependent mortality. In each case we study whether, and to what extent, the model extension under consideration can solve the empirical deficiencies of the core model. Both individual decisions and (simulated) general equilibrium effects are studied.

2.4.1 Hump-shaped productivity

In this section we directly address empirical deficiency (ED3) and assume that labour productivity of individuals is hump-shaped. That is, labour productivity is non-negative throughout life, starts out positive, is rising during the first life phase, and declines thereafter. For ease of exposition and future reference we collect the results concerning the individual labour productivity profile in Box 2.1 below.

The production side of the model is affected as follows. The total stock of efficiency

(i) Labour productivity of an u-year old is given by:

$$E(u) = \alpha_0 e^{-\zeta_0 u} - \alpha_1 e^{-\zeta_1 u}. \tag{B1.1}$$

we assume that $\alpha_0 > \alpha_1 > 0$, $\zeta_1 > \zeta_0 > 0$, and $\alpha_1\zeta_1 > \alpha_0\zeta_0$.

(ii) We easily find that:

$$E(0) = \alpha_0 - \alpha_1 > 0, \qquad \lim_{u\to\infty} E(u) = 0, \tag{B1.2}$$

$$E'(u) = -\zeta_0\alpha_0 e^{-\zeta_0 u} + \beta_1\alpha_1 e^{-\zeta_1 u} \begin{cases} > 0 & \text{for } 0 \le u < \bar{u} \\ < 0 & \text{for } u \ge \bar{u} \end{cases} \tag{B1.3}$$

where $\bar{u}$ is the age at which labour productivity is at its maximum:

$$\bar{u} = \frac{1}{\zeta_1 - \zeta_0} \ln\left(\frac{\alpha_1\zeta_1}{\alpha_0\zeta_0}\right). \tag{B1.4}$$

(iii) Along the balanced growth path the wage of an u-year old is given by (see below):

$$w(u) = w(0)\, e^{\gamma u}\left[\alpha_0 e^{-\zeta_0 u} - \alpha_1 e^{-\zeta_1 u}\right], \tag{B1.5}$$

Box 2.1: Labour Productivity Profile

units of labour at time t is denoted by $N(t)$ and is defined in the usual way:

$$N(t) \equiv \int_{-\infty}^{t} P(v,t)\, E(t-v)\, L(v,t)\, dv, \tag{2.37}$$

where $L(v,t)$ stands for raw labour supply in hours, and $P(v,t)$ is the size of cohort v at time t. Replacing L_i by N_i in equation (2.1), and redefining $\kappa_i \equiv K_i/N_i$ and $\kappa \equiv K/N$, we find that (2.5) and (2.7) are still satisfied but (2.6) must be changed to:

$$w(t)\, N(t) = (1 - \varepsilon_K)\, Y(t), \tag{2.38}$$

where $w(t)$ stands for the rental rate on efficiency units of labour. The wage faced at time t by a worker born at time v is thus given by:

$$w(v,t) \equiv E(t-v)\, w(t). \tag{2.39}$$

The household side of the model is affected as follows. In the household budget identity (2.9), $w(\tau)$ is replaced by $w(v,\tau)$. Along the balanced growth path, $w(v,\tau)$ can be written as:

$$w(v,\tau) = w(v)\, e^{\gamma(\tau - v)} \left[\alpha_0 e^{-\zeta_0(\tau-v)} - \alpha_1 e^{-\zeta_1(\tau-v)}\right], \tag{2.40}$$

where we have used (B1.1) and (2.39).[9] The consumption Euler equation is still given by (2.12). Interestingly, with a hump-shaped wage profile, it may be optimal for the agent to delay labour market entry somewhat. Indeed, we now have two relevant dates for the working decision of an agent, namely the optimal labour market entry date, S, and the optimal retirement date, R.[10] Obviously, we must have that $R > S \geq 0$. During working life ($S \leq \tau - v \leq R$) the condition (2.13) still holds but with $w(v,\tau)$ replacing $w(\tau)$.

Scaled consumption of a newborn agent is given by:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C + (1-\varepsilon_C)\left[e^{-(\rho+\mu)S} - e^{-(\rho+\mu)R}\right]} \cdot \frac{LI(v,v)}{w(v)}, \tag{2.41}$$

where $LI(v,v)/w(v)$ is defined as:

$$\frac{LI(v,v)}{w(v)} \equiv \int_S^R E(s)\, e^{-(r-\gamma+\theta\mu)s} ds + \frac{z}{r-\gamma+\theta\mu}. \tag{2.42}$$

For an interior solution (with $S > 0$), the labour market entry condition is given by:[11]

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S)\, e^{-[r-\gamma-\rho-(1-\theta)\mu]S}, \tag{2.43}$$

[9] Equation (2.40) shows that it is *in principle* possible for the individual's wage to fall after a certain age, namely if the fall in labour productivity exceeds the macroeconomic growth rate ($\dot{E}(u)/E(u) < -\gamma$). This effect does not occur in our calibrated model so the wage path is monotonically increasing in age.

[10] As was the case in the core model of the previous section, household preferences and the redistribution scheme are such that S and R are generation independent, i.e. $S(v) = S$ and $R(v) = R$ for all v.

[11] It is not difficult to show that an interior solution for S is obtained if the following condition is satisfied:

$$\frac{C(v,v)}{w(v)} > \frac{\varepsilon_C}{1-\varepsilon_C} E(0).$$

If this condition is violated, then $L(v,v)$ attains an interior solution satisfying:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(0)\left[1 - L(v,v)\right].$$

whereas the retirement condition is given by:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R)\, e^{-[r-\gamma-\rho-(1-\theta)\mu]R}. \tag{2.44}$$

Equations (2.41) (with (2.42) substituted), (2.43), and (2.44) form a three-equation system with three unknowns, viz. $C(v,v)/w(v)$, S, and R (see Table 3(a)). This system can be solved conditional on the macroeconomic variables, γ and z.

Using cross-section efficiency data for male workers aged between 18 and 70 from Hansen (1993, p. 74) we find the solid pattern in Figure 5(a). We interpolate these data by fitting equation (B1.1) using non-linear least squares. We find the following estimates (t-statistics in brackets): $\alpha_0 = 4.494$ (fixed), $\hat{\alpha}_1 = 4.010$ (71.04), $\hat{\zeta}_0 = 0.0231$ (24.20), $\hat{\zeta}_1 = 0.050$ (17.81) and the $R^2 = 0.80$ The fitted productivity profile is illustrated with dashed lines in Figure 5(a).

We have collected the key equations of the macroeconomic growth model in Table 3. Effectively this table provides the hump-shaped productivity analogue to Table 1. Compared to Table 1, the main changes are as follows. First, there is an additional equation governing the entry decision of households. Second, total labour supply is measured in efficiency units (i.e. n rather than l features in (T3.5)–(T3.7)). Third, the labour productivity age profile features prominently in (T3.2)–(T3.3) and (T3.7). The key features of the initial steady-state growth path have been reported in the first column of Table 2(b).

Figures 5(b)–(d) provide a visualization of the extended model. The key panel to consider is 5(b), which shows that with a hump-shaped productivity profile, the labour supply profile also features a hump-shaped pattern. This model extension thus somewhat alleviates empirical deficiency (ED2) of the core model. That is, we now have a labour supply profile that increases rapidly in young age, briefly touches a plateau and then drops to zero (i.e. retirement) quickly. Interestingly, the remaining empirical deficiencies (ED1) and (ED4)–(ED5) are not solved by the introduction of age dependent labour productivity. Consumption and assets are not hump shaped, and the required capital efficiency parameter, though lower than for the core model, is still too high ($\varepsilon_K = 0.6963$).

As before, the dashed lines in Figures 5(b)–(d) visualize the implications of an im-

Table 2.3. Balanced growth and retirement with age-dependent productivity

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C(\rho+\mu)}{\varepsilon_C+(1-\varepsilon_C)\left[e^{-(\rho+\mu)S}-e^{-(\rho+\mu)R}\right]} \cdot \left[\int_S^R E(s)\, e^{-(r-\gamma+\theta\mu)s} ds + \frac{z}{r-\gamma+\theta\mu}\right] \quad \text{(T3.1)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S)\, e^{-(r-\gamma-\rho-(1-\theta)\mu)S} \quad \text{(T3.2)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R)\, e^{-(r-\gamma-\rho-(1-\theta)\mu)R} \quad \text{(T3.3)}$$

(b) Macroeconomic relationships:

$$z = \mu(1-\theta)\frac{k(t)}{w(t)} \quad \text{(T3.4)}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r-\pi+\left[n-\frac{c(t)}{w(t)}\right]\cdot\frac{w(t)}{k(t)} \quad \text{(T3.5)}$$

$$\frac{w(t)\,n}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \quad \text{(T3.6)}$$

$$n \equiv \int_S^R \beta E(s)\, e^{-\beta s} ds - \beta e^{-\beta R} E(R)\, \frac{e^{[\gamma+\beta+\rho+(1-\theta)\mu-r](R-S)}-1}{\gamma+\beta+\rho+(1-\theta)\mu-r} \quad \text{(T3.7)}$$

$$\frac{c(t)}{w(t)} \equiv \frac{\beta}{\gamma+\beta+\rho+(1-\theta)\mu-r}\cdot\frac{C(v,v)}{w(v)} \quad \text{(T3.8)}$$

Definitions: Endogenous are: $C(v,v)/w(v)$, S, R, z, γ, n, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: birth rate β, mortality rate μ, population growth rate $\pi \equiv \beta-\mu$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K\Omega_0-\delta$, where δ is the depreciation rate of capital.

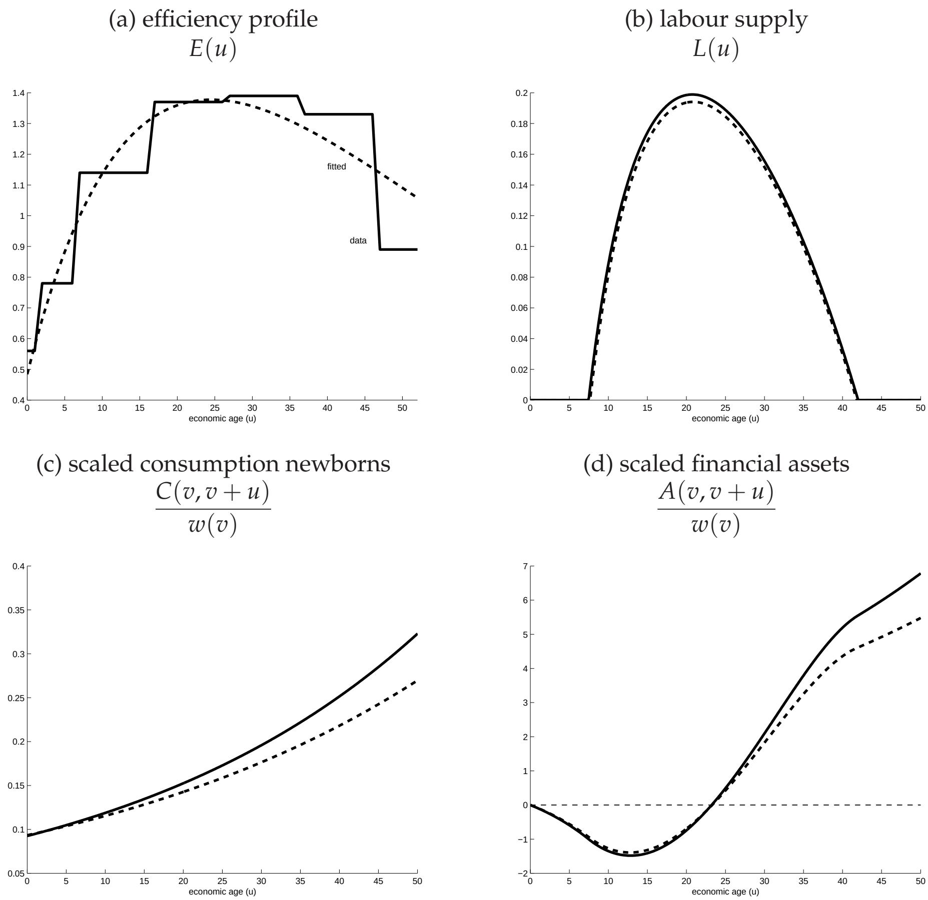

Figure 2.5. General equilibrium with age-dependent labour productivity

perfect annuity market (captured by $\theta = 0.7$). The key features of the new steady-state growth path have been reported in the second column of Table 2(b). The composite impact of an imperfect annuity market on individual decisions is that agents delay labour market entry, work less during working life and retire early. In general equilibrium this leads to a substantial reduction in economic growth. Interestingly, the effect on economic growth is very similar for the core model and the extended model.

Note that because the profits made by annuity firms are taxed away by the government, the annuity market imperfection acts as an implicit tax on the annuity premium. Furthermore, in the initial phase of the life-cycle the annuity market imperfection may act as a subsidy on loans. Although this is a somewhat troubling feature of the model, the effect of this feature is negligible because the absolute magnitude of loans as well as $\mu(t-v)$ are low for the young.

2.4.2 Age-dependent mortality

In this section we assume $E(u) = 1$ for all u and instead augment the core model by assuming age-dependent mortality. For ease of exposition, we use a demographic process which incorporates a finite maximum age; the Boucekkine, de la Croix, and Licandro (BCL) model suggested by Boucekkine *et al.* (2002). As with the labour productivity profile we collect the results concerning the mortality structure in Box 2.2 below.

We use data from age 18 onward for the Dutch cohort that was born in 1960. Following Heijdra and Romp (2008), we denote the actual surviving fraction up until model age u_i by S_i, and estimate the parameters of the parametric distribution function by means of non-linear least squares. The model to be estimated is thus:

$$S_i = 1 - \Phi(u_i) + \varepsilon_i = d\left(u_i \leq D\right) \cdot \frac{\eta_0 - e^{\eta_1 u_i}}{\eta_0 - 1} + \varepsilon_i, \tag{2.45}$$

where $d\left(u_i \leq \bar{D}\right) = 1$ for $u_i \leq \bar{D}$, and $d\left(u_i \leq \bar{D}\right) = 0$ for $u_i > \bar{D}$, and ε_i is the stochastic error term. We find the following estimates (with t-statistics in brackets): $\hat{\eta}_0 = 122.643$ (11.14), $\hat{\eta}_1 = 0.0680$ (48.51). The standard error of the regression is $\hat{\sigma} = 0.02241$, and the implied estimate for $\bar{D}$ is 70.75 model years (i.e., the maximum age in biological years

(i) The surviving fraction up to age u (from the perspective of birth) is given by:

$$1 - \Phi(u) \equiv \frac{\eta_0 - e^{\eta_1 u}}{\eta_0 - 1}, \tag{B2.1}$$

with $\eta_0 > 1$, $\eta_1 > 0$ and $\bar{D} = (1/\eta_1) \ln \eta_0$ is the maximum attainable age.

(ii) For $0 < s < \bar{D}$, the cumulative mortality rate is:

$$M(u) \equiv -\ln[1 - \Phi(u)], \tag{B2.2}$$

so that the exponential discounting factors are given by:

$$e^{-M(s)} \equiv \frac{\eta_0 - e^{\eta_1 s}}{\eta_0 - 1}, \qquad e^{M(s)} \equiv \frac{\eta_0 - 1}{\eta_0 - e^{\eta_1 s}}. \tag{B2.3}$$

(iii) The instantaneous mortality (or hazard) rate at age u is given by:

$$\mu(u) \equiv \frac{\Phi'(u)}{1 - \Phi(u)} = \frac{\eta_1 e^{\eta_1 u}}{\eta_0 - e^{\eta_1 u}}. \tag{B2.4}$$

The mortality rate is increasing in age and becomes infinite at $u = \bar{D}$.

(iv) The relative cohort size is:

$$p(v,t) \equiv \frac{P(v,t)}{P(t)} \equiv \beta e^{-\pi(t-v) - M(t-v)} = \beta \frac{\eta_0 - e^{\eta_1 (t-v)}}{\eta_0 - 1} e^{-n(t-v)} \tag{B2.5}$$

where β is the crude birth rate and π is the population growth rate.

(v) The demographic steady-state is given by (see d'Albis (2007, p.416) and Heijdra and Romp (2008, p.94)):

$$\frac{1}{\beta} = \frac{1}{\eta_0 - 1} \left[\eta_0 \frac{1 - e^{-n\bar{D}}}{n} + \frac{1 - e^{(\eta_1 - n)\bar{D}}}{\eta_1 - n} \right] \tag{B2.6}$$

For a given birth rate, equation (B2.6) determines the unique population growth rate consistent with the demographic steady state. The average population-wide mortality rate, $\bar{\mu}$, follows residually from the fact that $\pi \equiv \beta - \bar{\mu}$.

Box 2.2: Boucekkine, de la Croix and Licandro (2002) Mortality Structure

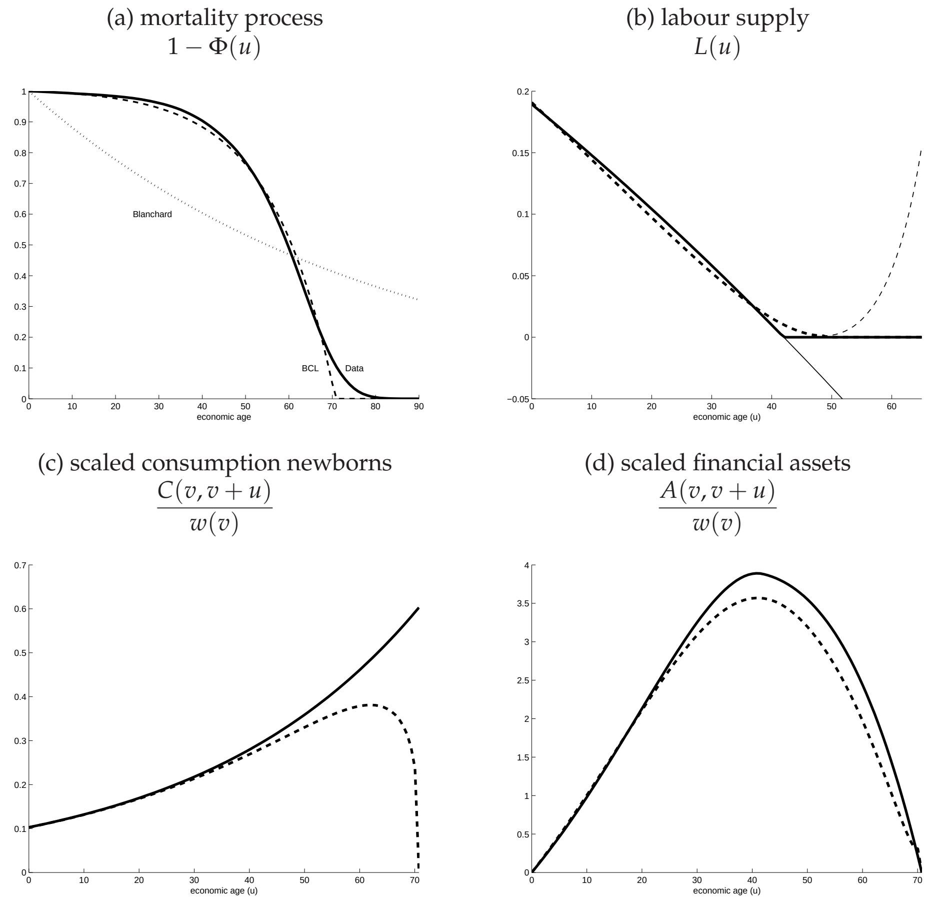

Figure 2.6. General equilibrium with age-dependent mortality

is 88.75). Figure 6(a) depicts the actual and fitted survival rates with, respectively, solid and dashed lines. Up to age 69, the BCL model fits the data rather well. For higher ages the fit deteriorates as the BCL model fails to capture the fact that some people are expected to live to very ripe old ages in reality.

Using the same data, we also estimate the parameter of the Blanchard demography, by running the following regression by means of non-linear least squares: $S_i = e^{-\mu u_i} + \varepsilon_i$. We find $\hat{\mu} = 0.0126$ (11.41), and $\hat{\sigma} = 0.2466$. The dotted line in Figure 6a depicts the fitted survival rates implied by the Blanchard demography. The fit is much worse than that of the BCL model. Relative to the data, the Blanchard model "kills off" the young too quickly and the old too slowly.

In the presence of age-dependent mortality, the core model is changed as follows. First, as is explained in Heijdra and Romp (2008, p. 92), the lifetime utility function (2.8) is now given by:

$$\mathbb{E}\Lambda\left(v,t\right) \equiv e^{M(t-v)} \cdot \int_{t}^{v+\bar{D}} \ln\left[C(v,\tau)^{\varepsilon_C} \cdot \left[1-L(v,\tau)\right]^{(1-\varepsilon_C)}\right] \cdot e^{-\rho(\tau-t)-M(\tau-v)} d\tau, \tag{2.46}$$

where (a) the maximum possible age is incorporated in the upper limit of the integral, and (b) the discounting factor due to lifetime uncertainty, $e^{-M(\tau-v)}$, depends on the agent's age at some future time τ.

Second, the annuity rate given in (2.10) above is modified to reflect the fact that the mortality rate depends on age:

$$r^{A}\left(\tau-v\right) \equiv r+\theta\mu\left(\tau-v\right), \qquad \text{(for } 0 \leq \tau-v < \bar{D}\text{)}. \tag{2.47}$$

Older agents attract a higher annuity rate than younger agents do because they feature a higher mortality rate (note that at age $\tau - v = \bar{D}$ no life insurance is available). Utility maximization gives rise to an individual consumption Euler equation that is different from the one given in (2.12) above:

$$\frac{\dot{C}\left(v,\tau\right)}{C\left(v,\tau\right)} = r-\rho-\left(1-\theta\right)\mu\left(\tau-v\right). \tag{2.48}$$

Provided annuities are imperfect ($\theta < 1$), optimal consumption growth is age dependent.

The key expressions characterizing individual behaviour are given in equations (T4.1)–(T4.3) in Table 2.4. Equation (T4.1) gives the expression for scaled consumption at birth. It contains specific values for a general demography-dependent function that is defined as follows:

$$\Xi\left(\lambda_1,\lambda_2\right)_{u_0}^{u_1} = \int_{u_0}^{u_1} e^{-\lambda_1 s} \cdot \left[\frac{\eta_0-e^{\eta_1 s}}{\eta_0-1}\right]^{\lambda_2} ds, \tag{2.49}$$

with $0 \leq u_0 < u_1 \leq \bar{D}$ and $\lambda_2 \geq 0$. Provided λ_1 is finite, the integral exists and is strictly positive. It follows that $\Xi\left(r-\gamma,\theta\right)_S^R > 0$, $\Xi\left(r-\gamma,\theta\right)_0^{\bar{D}} > 0$, $\Xi\left(\rho,1\right)_S^R > 0$, and $\Xi\left(\rho,1\right)_0^{\bar{D}} > 0$, so scaled newborn consumption is positive and depends positively on the amount of transfers.[12]

Interestingly, despite the fact that productivity is age-independent, equation (T4.2) shows that with imperfect annuities it is *in principle* possible for the individual agent to postpone labour market entry somewhat, i.e. to choose $S > 0$. With a realistic demography, however, this scenario does not materialize, i.e. *in practice* labour market entry is immediate and $S = 0$. Intuitively, this results from the fact that the mortality process only cuts in toward the end of the agent's life.

The macroeconomic part of the model is given by equations (T4.5)–(T4.9) in Table 2.4. Compared to the core model, the main changes are found in (T4.5) and (T4.8)–(T4.9). In (T4.5), transfers can no longer be related to a single aggregate variable but must be computed (numerically) by using the scaled wealth paths of existing cohorts. Expressions (T4.8)–(T4.9) generalize (T1.6)–(T1.7), making use of the $\Xi\left(\lambda_1,\lambda_2\right)_{u_0}^{u_1}$ function defined in (2.49) above.

Just as for the previous two models, we calibrate the model for an initial steady state with perfect annuities ($\theta = 1$), a growth rate of two percent ($\gamma = 0.02$), and an optimal retirement age of 42 years ($R = 42$). The key features of the initial steady-state growth path have been reported in the first column of Table 2(c). As was mentioned above, labour market entry is immediate for the cases considered in Table 2(c).

Figures 6(b)–(d) provide a visualization of the extended model. The key panels to consider are 6(c) and 6(d). With imperfect annuities, consumption features a hump-shaped pattern thus addressing empirical deficiency (ED1)–see the dashed lines in Figure 6(c). This finding is in line with Yaari (1965), Abel (1985), Bütler (2001), and Hansen and İmrohoroğlu (2008): with imperfect annuities the mortality rate features in the individual Euler equation. Hence, if the mortality rate is age-dependent, agents will discount consumption later on in life more heavily, thus creating a hump-shaped profile. From an empirical point of view it should be noted that we–like Bütler (2001) and Hansen and İmrohoroğlu (2008)–also find that the hump occurs too late in life. Also, as is illustrated in Figure 6(d), financial assets feature a hump-shaped pattern

[12] Using the Ξ-function we can define the demographic steady-state as $\frac{1}{\beta} = \Xi\left(\pi,1\right)_0^{\bar{D}}$ which simply generalizes (2.21) to the case with age-dependent mortality.

Table 2.4. Balanced growth and retirement with age-dependent mortality

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C \Xi(r-\gamma,\theta)_S^R + \varepsilon_C \Xi(r-\gamma,\theta)_0^{\bar{D}} \cdot z}{(1-\varepsilon_C)\,\Xi(\rho,1)_S^R + \varepsilon_C \Xi(\rho,1)_0^{\bar{D}}} \tag{T4.1}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho)S+(1-\theta)M(S)} \tag{T4.2}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} e^{-(r-\gamma-\rho)R+(1-\theta)M(R)} \tag{T4.3}$$

$$\frac{A(v,v+u)}{w(v)} \cdot e^{-ru-\theta M(u)} = -\frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_0^u + z \cdot \Xi(r-\gamma,\theta)_0^u \tag{T4.4a}$$

$$= -\frac{C(v,v)}{w(v)} \cdot \left[\Xi(\rho,1)_0^S + \frac{1}{\varepsilon_C}\Xi(\rho,1)_S^u\right] + z \cdot \Xi(r-\gamma,\theta)_0^u + \Xi(r-\gamma,\theta)_S^u \tag{T4.4b}$$

$$= \frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_u^{\bar{D}} - z \cdot \Xi(r-\gamma,\theta)_u^{\bar{D}} \tag{T4.4c}$$

(b) Macroeconomic relationships:

$$z = (1-\theta) \cdot \int_0^{\bar{D}} \beta e^{-(\pi+\gamma)u-M(u)} \mu(u) \frac{A(v,v+u)}{w(v)} du \tag{T4.5}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[l - \frac{c(t)}{w(t)}\right] \cdot \frac{w(t)}{k(t)} \tag{T4.6}$$

$$\frac{w(t)\,l}{k(t)} = (1-\varepsilon_K)\,\Omega_0 \tag{T4.7}$$

$$l = \beta \cdot \left[\Xi(\pi,1)_S^R - \frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v)}{w(v)} \cdot \Xi(\pi+\rho+\gamma-r,2-\theta)_S^R\right] \tag{T4.8}$$

$$\frac{c(t)}{w(t)} \equiv \frac{C(v,v)}{w(v)} \cdot \beta\Xi(\pi+\rho+\gamma-r,2-\theta)_0^{\bar{D}} \tag{T4.9}$$

Definitions: Endogenous are $C(v,v)/w(v)$, S, R, z, γ, l, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: birth rate β, aggregate mortality rate $\bar{\mu}$, population growth rate $\pi \equiv \beta - \bar{\mu}$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K \Omega_0 - \delta$, where δ is the depreciation rate of capital.

both with perfect and with imperfect annuities.[13] The model extension thus fixes empirical deficiency (ED4) to a large extent. Finally, empirical deficiency (ED5) is reduced somewhat in this extension as the required efficiency parameter for capital is equal to $\varepsilon_K = 0.6956$ (rather than 0.8348 in the core model).

As before, the dashed lines in Figures 6(b)–(d) visualize the implications of an imperfect annuity market (captured by $\theta = 0.7$). The key features of the new steady-state growth path have been reported in the second column of Table 2(c). Just as in the core model, individual and aggregate saving and thus the macroeconomic growth rate are all lower when annuity markets are imperfect rather than perfect.[14] Furthermore, and in contrast to both the core model and the model with age-dependent productivity, we now find that agents also delay labour market exit by almost seven years. Hence, the composite impact of an imperfect annuity market on individual decisions is that agents work slightly fewer hours during most of their working life, but retire much later thus limiting the fall in the aggregate supply of labour. In general equilibrium, this retirement effect explains why the reduction in economic growth is smaller than for the previous two models.

Compared to the core model, the annuity market imperfection operates quite differently in the model with a realistic demographic structure – compare panels (a) and (c) in Table 2. First, instead of finding a near-zero retirement effect, in the extended model agents *delay* retirement by almost 7 years. Intuitively, with age-dependent mortality and imperfect annuities, agents discount future felicity by their ever increasing mortality rate. This ensures that both consumption and leisure are hump-shaped and hence labour supply is U-shaped. Because retirement is an absorbing state, however, the upward sloping part of the labour supply path is not attained – see Figure 6(b).[15] In the calibration underlying Table 2(c), labour supply bottoms out at zero thus explaining the large shift in the retirement age.

Second, instead of experiencing a reduction in the economic growth rate of 35 basis

[13] In contrast to the core model, with age-dependent mortality an actuarially fair redistribution scheme of the form $TR(v,\tau) = (1-\theta)\,\mu(\tau - v)\,A(v,\tau)$ is feasible. See also footnote 6 on this issue.

[14] This finding regarding growth has previously been highlighted by Abel (1985) and Fuster (1999) who suggest that capital accumulation decreases with imperfect annuities provided (i) the elasticity of intertemporal substitution is no less than unity and (ii) there is steady-state growth.

[15] In Figure 6(b) the thin dotted line after retirement gives the labour supply path if retirement were not an absorbing state. Similar, the thin solid line gives labour supply if negative labour supply were allowed.

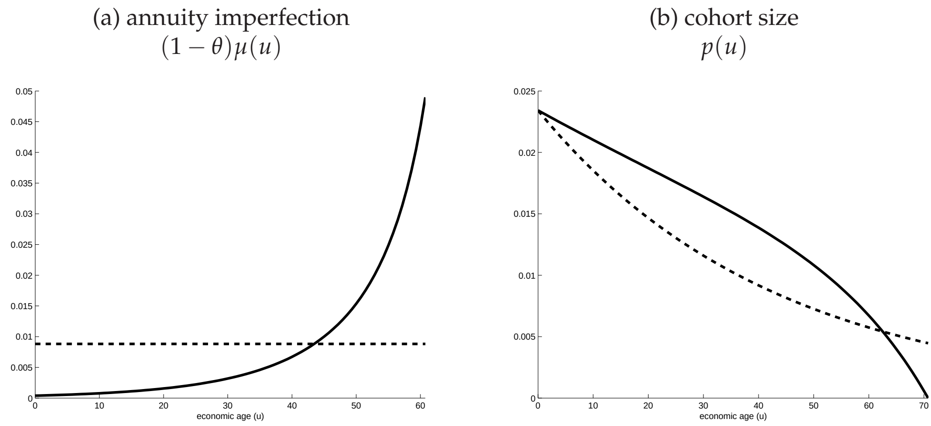

Figure 2.7. The annuity market imperfection

points, in the extended model we see a much smaller reduction of only 8 basis points. To appreciate the origin of these differences, note Figures 7(a)–(b). Figure 7(a) visualizes the annuity market imperfection faced by the agent over the life-cycle. The dashed line shows the imperfection for the Blanchard mortality process whilst the solid line depicts the imperfection for the realistic-mortality case. From here it is immediately clear that the Blanchard mortality process overstates the magnitude of the annuity market imperfection for a substantial part of the life-cycle. In contrast, for a realistic demography the annuity market imperfection only becomes an issue later on in life. Furthermore, as can be seen in Figure 7(b), the relative size of cohorts that are actually affected is quite small. In the core model assets grow indefinitely with age, thus overstating the effect of older agents on the growth rate. A given change in θ thus has a large effect on growth because it most strongly affects the asset-rich older agents of whom there are too many. In the extended model, however, these agents are not only relatively few in numbers but are also decumulating assets. As a result, they have a much smaller effect on the growth rate.

2.4.3 Full model

In this section we visualize the full model, simultaneously incorporating age-dependent labour productivity and mortality. The key equations for the full model have been collected in Table 2.5, whilst Figure 8 visualizes some of its salient life-cycle features. Finally, the quantitative effect of imperfect annuities are reported in Table 2(d).

Figure 8(a) plots the right-hand sides of (T5.2) and (T5.3) as a function of age. For $\theta = 1$, there is a unique entry age ($S = 7.47$, at point A) and a unique retirement age ($R = 42$, at point B). In contrast, for $\theta = 0.7$, there appears to be a second labour market entry point located to the right of point B. This point is not feasible, however, because we assume that labour market exit is an absorbing state. Hence, also for $\theta = 0.7$, there are unique entry and exit ages, i.e. $S = 7.40$ and $R = 42.40$ – see Table 2(d).

Figure 8(b) shows the age profile for labour supply. It is hump-shaped because labour productivity is, i.e. Figure 8(b) looks very much like Figure 5(b) above.

Figure 8(d) shows the age profile for financial assets. This figures captures the main features of Figure 6(d), but adds a borrowing period at the start of life. Agents delay labour market entry and –upon entry– face rather low wages and supply few hours early on in life. They finance their rising consumption profile by borrowing during that first life phase.

Interestingly, the quantitative effects of θ on the optimal retirement age and growth are rather small, as is revealed in column (i) of Table 2(d). The full model is a hybrid of the two extended models. With respect to the optimal retirement decision, the effects explained by age-dependent productivity outweigh the effects of age-dependent mortality. In contrast, the impact of θ on the growth rate is predominantly driven by the effects of age-dependent mortality. Furthermore, empirical deficiency (ED5) is eliminated in the full model as the required efficiency parameter for capital is equal to $\varepsilon_K = 0.2402$.

2.4.4 The role of transfers in the full model

Up until now we have focused on the situation where the profits made by the annuity firms are redistributed toward the agents in the form of a lump-sum transfer. These transfers have allowed us to focus solely on the substitution effect of the annuity market imperfection. However, in order to study the full (i.e. income and substitution) effect of the imperfection we need to consider an alternative general equilibrium mechanism by which the profits of the annuity firms are spent. In this subsection we assume

Table 2.5. Balanced growth and retirement with age-dependent productivity and mortality

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{\alpha_0 \varepsilon_C \Xi(r+\zeta_0-\gamma,\theta)_S^R - \alpha_1 \varepsilon_C \Xi(r+\zeta_1-\gamma,\theta)_S^R}{(1-\varepsilon_C)\Xi(\rho,1)_S^R + \varepsilon_C \Xi(\rho,1)_0^{\bar{D}}} + \frac{\varepsilon_C \Xi(r-\gamma,\theta)_0^{\bar{D}} \cdot z}{(1-\varepsilon_C)\Xi(\rho,1)_S^R + \varepsilon_C \Xi(\rho,1)_0^{\bar{D}}} \quad \text{(T5.1)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S)\, e^{-(r-\gamma-\rho)S+(1-\theta)M(S)} \quad \text{(T5.2)}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R)\, e^{-(r-\gamma-\rho)R+(1-\theta)M(R)} \quad \text{(T5.3)}$$

$$\frac{A(v,v+u)}{w(v)} \cdot e^{-ru-\theta M(u)} = -\frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_0^u + z \cdot \Xi(r-\gamma,\theta)_0^u \quad \text{(T5.4a)}$$

$$= -\frac{C(v,v)}{w(v)} \cdot \left[\Xi(\rho,1)_0^S + \frac{1}{\varepsilon_C}\Xi(\rho,1)_S^u\right] + z \cdot \Xi(r-\gamma,\theta)_0^u + \left[\alpha_0 \Xi(r+\zeta_0-\gamma,\theta)_S^u - \alpha_1 \Xi(r+\zeta_1-\gamma,\theta)_S^u\right] \quad \text{(T5.4b)}$$

$$= \frac{C(v,v)}{w(v)} \cdot \Xi(\rho,1)_u^{\bar{D}} - z \cdot \Xi(r-\gamma,\theta)_u^{\bar{D}} \quad \text{(T5.4c)}$$

(b) Macroeconomic relationships:

$$z = (1-\theta) \cdot \int_0^{\bar{D}} \beta e^{-(\pi+\gamma)u - M(u)} \mu(u) \frac{A(v,v+u)}{w(v)} du \quad \text{(T5.5)}$$

$$\gamma \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[n - \frac{c(t)}{w(t)}\right] \cdot \frac{w(t)}{k(t)} \quad \text{(T5.6)}$$

$$\frac{w(t)\, n}{k(t)} = (1-\varepsilon_K)\Omega_0 \quad \text{(T5.7)}$$

$$n = \beta \cdot \left[\alpha_0 \Xi(\pi+\zeta_0,1)_S^R - \alpha_1 \Xi(\pi+\zeta_1,1)_S^R - \frac{1-\varepsilon_C}{\varepsilon_C} \frac{C(v,v)}{w(v)} \cdot \Xi(\pi+\rho+\gamma-r, 2-\theta)_S^R\right] \quad \text{(T5.8)}$$

$$\frac{c(t)}{w(t)} \equiv \frac{C(v,v)}{w(v)} \cdot \beta \Xi(\pi+\rho+\gamma-r, 2-\theta)_0^{\bar{D}} \quad \text{(T5.9)}$$

Definitions: Endogenous are $C(v,v)/w(v)$, S, R, z, γ, n, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: birth rate β, aggregate mortality rate $\bar{\mu}$, population growth rate $\pi \equiv \beta - \bar{\mu}$, imperfection annuities θ, rate of time preference ρ, capital coefficient in the technology ε_K, consumption coefficient in tastes ε_C, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon_K \Omega_0 - \delta$, where δ is the depreciation rate of capital.

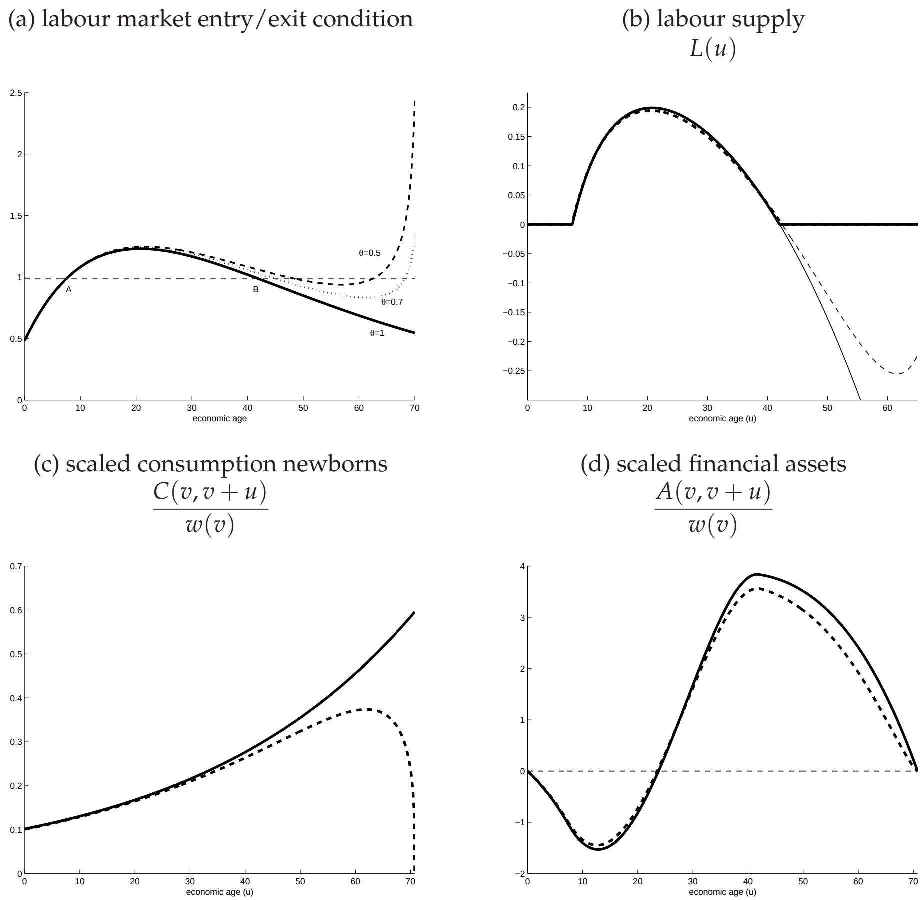

Figure 2.8. General equilibrium with age-dependent productivity and mortality

that the government uses the funds for non-productive spending.

Compared to Table 5, there are two major changes. First, transfers are zero both with perfect and imperfect annuities. Second, the imperfection surfaces directly in the relationship for the growth rate. Indeed, equation (T5.5) is replaced by:

$$\gamma = \frac{\dot{k}(t)}{k(t)} = r - \pi - (1-\theta) \cdot \frac{w(t)\,\Gamma}{k(t)} + \left[n - \frac{c(t)}{w(t)}\right] \frac{w(t)}{k(t)}, \tag{T5.5$'$}$$

where $w(t)\,\Gamma$ is given by:

$$w(t)\,\Gamma \equiv \int_{t-\bar{D}}^{t} p(v,t)\,\mu(t-v)\,A(v,t)\,dv.$$

Figure 9 visualizes the impact of the annuity market imperfection on labour supply and financial assets (as in the full model with transfers, consumption is hump shaped). Figure 9(b) shows that assets accumulation is increased slightly for younger agents and reduced substantially for older agents.

Comparing columns (i) and (ii) two main features stand out. First, the growth rate drops substantially under non-productive government spending. This is a direct consequence of draining productive resources from the economy. Second, in the model with unproductive spending agents enter and retire at an earlier age and shorten the length of their working career. The retirement effect is a direct consequence of the fall in the growth rate – see for example the discussion surrounding Figure 2.

2.5 Conclusions

We study the impact of imperfect annuity markets on individual decisions and macroeconomic outcomes. We develop a concise overlapping generations model of a closed economy featuring endogenous growth. We demonstrate that this model replicates the most salient life-cycle features of asset holdings, labour supply, and consumption. For this, annuities must be imperfect and both the mortality process and labour productivity must be age dependent. The annuity imperfection accounts for a hump-shaped consumption profile, age-dependent mortality gives rise to a life-cycle pattern of saving, and age-dependent productivity captures the life-cycle pattern of labour supply.

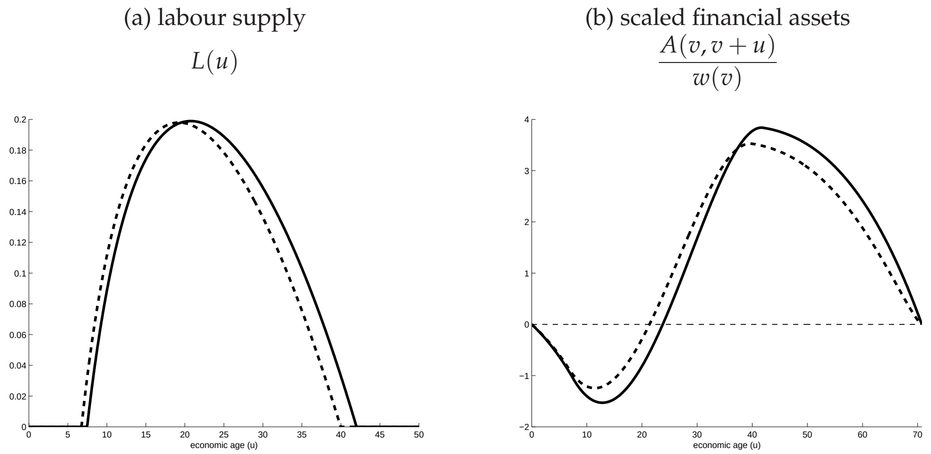

Figure 2.9. Full model with useless government spending

The empirical evidence suggests that the annuity imperfection parameter employed in this chapter may be quite substantial. Our model shows that the microeconomic effects of such an imperfection are rather large if the lump-sum transfers (arising from annuity firms' profits) are not taken into account. But this partial equilibrium result, though commonly stressed in the literature, is rather misleading. Indeed, in the presence of transfers, i.e. in a general equilibrium setting, both microeconomic and macroeconomic effects of quantitatively significant annuity imperfections are small.

The model developed in this chapter can be amended and extended in a straightforward fashion to study a variety of public policy issues. In the next chapter we use the model to study the implications of labour income and consumption taxation. In Chapter 4 we amend the model slightly and study how different public pension arrangements affect a decrease in adult mortality.

Chapter 3

Labour-income and consumption taxes*

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* This chapter is based on Heijdra and Mierau (2010).

3.1 Introduction

In this chapter we use the model developed in the previous chapter to revisit a classic theme in dynamic public finance theory, namely the effects of consumption and labour-income taxation on the macroeconomic allocation and long-run growth rate. Rather than studying corrective taxation aimed at internalizing the knowledge externality, we focus on a number of traditional public tax policy questions. How do the taxes affect individual decisions regarding consumption, labour supply and saving? And by which mechanisms is the macroeconomic growth rate affected? And to what extent does the method of tax revenue recycling affect the microeconomic decisions and the macroeconomic outcomes?

The government's budgetary policy plays a vital role in our analysis. We analyze three different revenue recycling methods via lump-sum transfers. The first method assumes that these transfers are the same for everybody, regardless of age. The second method gives higher transfers to the young, whereas the third method biases the transfers to favour the old. In order to demonstrate the vital importance of the revenue-recycling mechanism we also discuss the hypothetical scenario in which the government uses its tax revenue for useless government consumption expenditures.

Among other things, we find that tax/transfer combinations that redistribute funds away from the "dissaving elderly" toward the "saving young" leads to higher economic growth. Not surprisingly, wasteful government consumption has a disastrous effect on economic growth. In addition we find that a consumption tax positively dominates a labour-income tax, both in growth and in welfare terms.

The paper that comes closest to ours is Heijdra and Ligthart (2000) which studies the consequences of capital, labour, and consumption taxes on aggregate capital formation and welfare in a perpetual-youth overlapping-generations model of the Blanchard (1985) type. Their model highlights the importance of intergenerational redistribution effects of taxes. These effects are not present in the infinitely-lived representative-agent model but turn out to play a crucial role. That is, whereas representative-agent models predict an unambiguous negative effect of any form of taxation, Heijdra and Ligthart show that intergenerational transfers may redistribute funds toward individuals with a higher marginal propensity to save, thereby increasing the steady-state capital stock.

In a similar vein, Petrucci (2002) studies the impact of consumption taxation in a model akin to the Heijdra and Ligthart model but with endogenous rather than exogenous growth. He finds that the transfers arising from consumption taxation can induce higher growth due to intergenerational transfers. This result is in stark contrast to the remainder of the literature, which focuses on representative-agent models and finds that taxation, regardless of its base, reduces economic growth (Turnovsky, 2000) or is, at best, neutral (Stokey and Rebelo, 1995).

We thus extend both Petrucci (2002) (by endogenizing the labour supply and retirement decision) and Heijdra and Ligthart (2000) (by including a retirement decision and endogenizing the long-run growth rate). In addition, our model features realistic life-cycle features as mentioned above. In accordance with Petrucci we find that the intergenerational transfers arising from consumption taxation lead to higher economic growth. In line with Heijdra and Ligthart we find that consumption and labour taxation are not equivalent, as they are in the representative-agent model. In addition, we show that the equivalence breaks down not only because of demographic factors but also because old individuals retire. The non-equivalence between consumption and labour taxation assures that the intergenerational transfer effects of these two taxes differ substantially. In particular, for certain transfer schemes the positive growth effect found for consumption taxation fails to uphold for labour taxes.

The remainder of the chapter is set up as follows. The next section outlines the model. Sections 3 and 4 study consumption and labour taxes, respectively. The final section concludes.

3.2 Model

We use the model developed in Chapter 2 and extend it to include taxes on consumption and labour income and alternative redistribution schemes. We refer the reader to Chapter 2 for a complete description of the model and use the remainder of this section to outline the model and its the extensions.

3.2.1 Consumers

Individual behaviour

Expected remaining-lifetime utility of an individual born at time v is given by:

$$\mathbb{E}\Lambda\left(v,v\right)\equiv\int_{v}^{v+\bar{D}}\ln\left[C(v,\tau)^{\varepsilon_C}\cdot\left[1-L(v,\tau)\right]^{1-\varepsilon_C}\right]\cdot e^{-\rho(\tau-v)-M(\tau-v)}d\tau, \tag{3.1}$$

where $C\left(v,\tau\right)$ is consumption, $L\left(v,\tau\right)$ is labour supply (the time endowment is equal to unity), ρ is the pure rate of time preference, $\bar{D}$ is the maximum attainable age for the agent, and $e^{-M(\tau-v)}$ is the probability that the agent is still alive at some future time τ $(\geq v)$. Here, $M(\tau-v)\equiv\int_0^{\tau-v}\mu(s)ds$ stands for the cumulative mortality rate and $\mu\left(s\right)$ is the instantaneous mortality rate of an agent of age s.

The agent's budget identity is given by:

$$\dot{A}(v,\tau)=r^A\left(\tau-v\right)A(v,\tau)+w(v,\tau)\left(1-\theta_L\right)L\left(v,\tau\right)-\left(1+\theta_C\right)C(v,\tau)+TR\left(v,\tau\right), \tag{3.2}$$

where $A\left(v,\tau\right)$ is the stock of financial assets, $r^A\left(\tau-v\right)$ is the age-dependent annuity rate of interest rate, $w\left(v,\tau\right)\equiv E\left(\tau-v\right)w\left(\tau\right)$ is the age-dependent wage rate, $E\left(\tau-v\right)$ is exogenous labour productivity, θ_L is the labour income tax, θ_C is the consumption tax, $TR\left(v,\tau\right)$ are lump-sum transfers (see below). As in Chapter 2 our model contains a number of distinguishing features, which we briefly summarize below.

Feature 1. We postulate the existence of annuity markets, but we allow the annuities to be less than actuarially fair. Since the agent is subject to lifetime uncertainty and has no bequest motive, he/she will fully annuitize so that the annuity rate of interest facing the agent is given by:

$$r^A\left(\tau-v\right)\equiv r+\lambda\mu\left(\tau-v\right),\qquad(\text{for }0\leq\tau-v<\bar{D}). \tag{3.3}$$

where r is the real interest rate, and λ is a parameter ($0<\lambda\leq1$) indicating the degree of imperfection on the annuity market.

Feature 2. We assume that labour productivity is hump-shaped over the life-cycle. This assures that labour supply is similarly hump-shaped over the life-cycle. A useful parameterization of the productivity profile is:

$$E(t-v) = \alpha_0 e^{-\zeta_0(t-v)} - \alpha_1 e^{-\zeta_1(t-v)}, \qquad (\text{for } 0 \le t-v \le \bar{D}), \tag{3.4}$$

where α_i and ζ_i are the parameters governing the curvature of the productivity profile (see Box 2.1 for details).

Feature 3. Following Boucekkine *et al.* (2002) we assume that $e^{-M(t-v)}$ takes the following rather convenient functional form:

$$e^{-M(t-v)} \equiv \frac{\eta_0 - e^{\eta_1(t-v)}}{\eta_0 - 1}, \qquad (\text{for } 0 \le t-v \le \bar{D}), \tag{3.5}$$

where $\eta_0 > 1$ and $\eta_1 > 0$ are parameters and $\bar{D}$ is the maximum attainable age (see Box 2.2 for details).

The agent chooses time profiles for $C(v,\tau)$, $A(v,\tau)$, and $L(v,\tau)$ (for $v \le \tau \le v + \bar{D}$) in order to maximize (3.1), subject to (i) the budget identity (3.2), (ii) a transversality condition, $A(v, v+\bar{D}) = 0$, (iii) the initial asset position at birth, $A(v,v) = 0$, and (iv) a non-negativity condition for labour supply, $L(v,\tau) \ge 0$. The solution of this optimization problem is fully characterized by the following equations:

$$C(v,t) = C(v,v) \cdot e^{(r-\rho)(t-v)-(1-\lambda)M(t-v)}, \tag{3.6}$$

$$\frac{(1-\varepsilon_C)/(1-L(v,t))}{\varepsilon_C/C(v,t)} = \frac{1-\theta_L}{1+\theta_C} \cdot w(v,t) \qquad (\text{for } S \le t-v \le R), \tag{3.7}$$

$$L(v,t) = 0 \qquad (\text{for } 0 \le t-v \le S \text{ and } R \le t-v \le \bar{D}), \tag{3.8}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S) \frac{1-\theta_L}{1+\theta_C} e^{-(r-\rho-\gamma)S+(1-\lambda)M(S)}, \tag{3.9}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R) \frac{1-\theta_L}{1+\theta_C} e^{-(r-\rho-\gamma)R+(1-\lambda)M(R)}, \tag{3.10}$$

$$(1+\theta_C) \cdot \frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{(1-\varepsilon_C)\int_S^R e^{-\rho s - M(s)} ds + \varepsilon_C \int_0^{\bar{D}} e^{-\rho s - M(s)} ds} \cdot \frac{H(v,v)}{w(v)}, \tag{3.11}$$

$$\frac{H(v,v)}{w(v)} \equiv (1-\theta_L) \int_S^R E(s) e^{-(r-\gamma)s - \lambda M(s)} ds + \int_0^{\bar{D}} \frac{TR(v, v+s)}{w(v+s)} e^{-(r-\gamma)s-\lambda M(s)} ds, \tag{3.12}$$

where $H(v,v)$ is human wealth at birth and γ is macroeconomic growth rate. The intuition behind these expressions is as follows. Equation (3.6) is best understood by noting the consumption Euler equation resulting from utility maximization:

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = r - \rho - (1-\lambda)\,\mu(\tau - v)\,. \tag{3.13}$$

By using this expression, future consumption can be expressed in terms of consumption at birth as in (3.6). In the absence of an annuity market imperfection ($\lambda = 1$), consumption growth only depends on the gap between the interest rate and the pure rate of time preference. In contrast, with imperfect annuities, individual consumption growth is negatively affected by the mortality rate, a result first demonstrated for the case with $\lambda = 0$ by Yaari (1965, p. 143).

Equations (3.7)–(3.10) characterize the agent's labour supply plans during the life cycle. There are two critical ages in the worker's life cycle, namely the labour market entry age S and the retirement age R. During youth, for $0 \leq t - v \leq S$ the agent has not yet entered the labour market. Toward the end of life, for $R \leq t - v \leq \bar{D}$, the agent no longer works. During the working period, the agent equates the marginal rate of substitution between leisure and consumption to the wage rate at all times – see equation (3.7). The optimal labour market entry and retirement points are determined in, respectively (3.9) and (3.10).

The consumption-leisure choice over the life cycle is illustrated in Figure 2, where $\frac{C(v,v+u)}{w(v)}$ and $L(u)$ stand for, respectively, consumption (scaled by the wage rate at birth) and labour supply of the agent at age $u \equiv t - v$. To facilitate the discussion we assume that annuities are perfect ($\lambda = 1$) so that consumption grows monotonically over the life cycle. We show four moments in the agent's life. The initial choice at age $u = 0$ is at point E_0. The wage rate is low, leisure is cheap, and the agent faces a binding non-negativity constraint on labour supply. For $0 < u < S$, this constraint remains binding but the agent chooses an increasing path for consumption. This is the gradual move from E_0 to E_S.

At age $u = S$ the agent achieves a tangency between an indifference curve, $U_S = C(v,v+S)^{\varepsilon_C} \cdot [1 - L(v,v+S)]^{1-\varepsilon_C}$ and a "budget equation" $X(v,v+S) = C(v,v+S) + w(v,v+S) \cdot [1 - L(v,v+S)]$, where $X(v,\tau)$ is full consumption and, of course, $L(v,v+S) = 0$. Equation (3.9) describes point E_S in terms of the key economic variables in the mo-

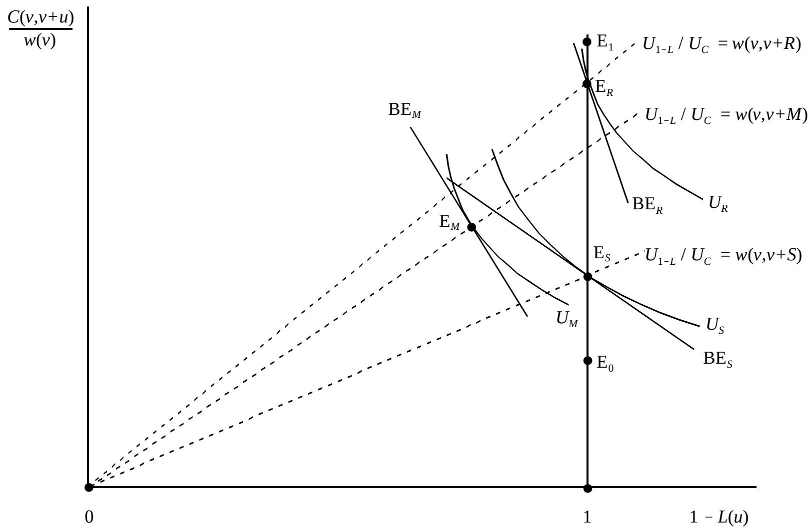


Figure 3.1. Life-cycle consumption and labour supply

del.

For $S < u < R$ the agent makes interior choices for both consumption and labour supply, and the optimum moves in north-westerly direction from point E_S. The wage increases with age but the substitution effect dominates the wealth effect and labour supply rises initially. During that life phase (and with perfect annuities), full consumption increases exponentially according to $\dot{X}(v, v+u) / X(v, v+u) = r - \rho > 0$. This causes the wealth effect to strengthen.

At some age $u = M$, the wealth effect exactly matches the substitution effect and labour supply reaches its peak. This occurs at point E_M in Figure 2. The wage at that point exceeds the wage at labour market entry, $w(v, v+M) > w(v, v+S)$, so the budget equation through E_M is steeper than the one through E_S. Beyond age M, the wealth effect dominates and labour supply falls gradually. In the parameterized version of our model, an individual's wage path is uniformly upward sloping (see Figure 3(d)) so the budget equation continues to rotate in a clockwise fashion as the agent gets older.

At age $u = R$, the agent retires from the labour market. Equation (3.10) describes point E_R in terms of the key economic variables in the model. The wage rate is high, leisure is expensive, but the agent is rather wealthy and thus faces a binding non-negativity constraint on labour supply, just as at the start of life but for diametrically opposite reasons. Beyond age R, consumption continues to increase. The optimum gradually moves from E_R in the direction of point E_1 in Figure 1.

Equation (3.11) shows that scaled consumption of a newborn is proportional to human wealth. The marginal propensity to consume out of human wealth at birth is decreasing in the length of the agent's working career. Finally, equation (3.12) provides the definition of human wealth at birth. The first term on the right-hand side represents the present value of the time endowment during working life, using the growth-corrected annuity rate of interest for discounting. The later one retires, the larger is this term. The second term on the right-hand side of (3.12) is just the present value of transfers.

In the presence of an age-dependent mortality process, the following demography-dependent function is quite convenient as it shows up in various places in the model characterization:

$$\Xi\left(\xi_1, \xi_2\right)_{u_0}^{u_1} = \int_{u_0}^{u_1} e^{-\xi_1 s} \cdot \left[\frac{\mu_0 - e^{\mu_1 s}}{\mu_0 - 1}\right]^{\xi_2} ds, \tag{3.14}$$

with $0 \leq u_0 < u_1 \leq \bar{D}$ and $\xi_2 \geq 0$. Provided ξ_1 and ξ_2 are finite, the integral exists and is strictly positive.

Aggregate household behaviour

In this subsection we derive expressions for per-capita average consumption, labour supply, and saving. With age-dependent mortality the demographic steady-state equilibrium has the following features:

$$1 = \beta\Xi\left(\pi, 1\right)_0^{\bar{D}}, \tag{3.15}$$

$$p\left(v, t\right) \equiv \frac{P\left(v, t\right)}{P\left(t\right)} \equiv \beta e^{-\pi(t-v)-M(t-v)}, \tag{3.16}$$

where β is the crude birth rate, π is the growth rate of the population, $p\left(v, t\right)$ and $P\left(v, t\right)$ are, respectively, the relative and absolute size of cohort v at time $t \geq v$, and $P\left(t\right)$ is the population size at time t. The average population-wide mortality rate, $\bar{\mu}$, follows residually from the fact that $\pi \equiv \beta - \bar{\mu}$.

Using (3.16), we can define per-capita average values in general terms as:

$$b(t) \equiv \int_{-\infty}^{t} p(v,t) B(v,t) \, dv, \tag{3.17}$$

where $B(v,t)$ denotes the variable in question at the individual level, and $b(t)$ is the per capita average value of that same variable. Using (3.6) and (3.17), we find that per capita average consumption can be written as follows:

$$\frac{c(t)}{w(t)} = \frac{C(v,v)}{w(v)} \cdot \beta \Xi (\pi + \rho + \gamma - r, 2 - \lambda)_0^{\bar{D}}. \tag{3.18}$$

Efficiency units of labour of vintage $t - v$ are defined as $N(t-v) \equiv E(t-v) L(v,t)$. Using this expression, as well as (3.4), (3.6)–(3.7), and (3.17) we find the per capita average supply of efficiency units of labour:

$$n = \beta \int_S^R E(s) e^{-\pi s - M(s)} ds - \frac{C(v,v)}{w(v)} \frac{1-\varepsilon_C}{\varepsilon_C} \frac{1+\theta_C}{1-\theta_L} \beta \Xi (\pi + \rho + \gamma - r, 2 - \lambda)_S^R, \tag{3.19}$$

with $0 < n < \bar{n}$, where $\bar{n} \equiv \beta \int_0^{\bar{D}} E(s) e^{-\pi s - M(s)} ds$ is the maximum labour potential in the economy. The first term on the right-hand side of (3.19) provides the first mechanism by which n falls short of $\bar{n}$: agents only work during part of their lives. Prior to labour market entry and after retirement, they consume their unit time endowment in the form of leisure. The second composite term on the right-hand side of (3.19) represents the second mechanism by which n falls short of $\bar{n}$: during their productive career, workers never supply their full time endowment to the labour market.

Finally, using (3.17) we observe that per capita average assets are defined as $a(t) \equiv \int_{-\infty}^{t} p(v,t) A(v,t) \, dv$ so that its rate of change is:

$$\dot{a}(t) = \int_{t-\bar{D}}^{t} p(v,t) \dot{A}(v,t) \, dv - \int_{t-\bar{D}}^{t} [\pi + \mu(t-v)] A(v,t) \, dv, \tag{3.20}$$

where we have incorporated the fact that individual agents have zero financial assets at birth and at the maximum attainable age ($A(v,v) = A(v, v+\bar{D}) = 0$) and that the relative cohort size evolves over time according to $\dot{p}(v,t) = -[\pi + \mu(t-v)] p(v,t)$.

Using (3.2)–(3.3) in (3.20) we thus find:

$$\begin{aligned}\dot{a}(t) &= (r-\pi)\,a(t) + (1-\theta_L)\,w(t)\,n - (1+\theta_C)\,c(t) + \int_{t-\bar{D}}^{t} p(v,t)\,TR(v,t)\,dv \\ &\quad - (1-\lambda)\int_{t-\bar{D}}^{t} \mu(t-v)\,p(v,t)\,A(v,t)\,dv. \end{aligned} \tag{3.21}$$

3.2.2 Government

We assume that the government maintains continuous budget balance and does not use debt financing. Total per capita tax receipts are given by:

$$tax(t) \equiv \theta_C c(t) + \theta_L w(t)\,n + (1-\lambda)\int_{t-\bar{D}}^{t} \mu(t-v)\,p(v,t)\,A(v,t)\,dv, \tag{3.22}$$

where the last term on the right-hand side represents the excess profits of the annuity industry that are taxed away by the government. We write the per capita government budget constraint as follows:

$$tax(t) = g\cdot k(t) + \int_{t-\bar{D}}^{t} p(v,t)\,TR(v,t)\,dv, \tag{3.23}$$

where $g \cdot k(t)$ is useless government spending and g is a non-negative parameter.

By using (3.22)–(3.23) in (3.21) and noting that the capital market equilibrium condition is given by $a(t) = k(t)$, we find the macroeconomic accumulation equation for the per capita capital stock:

$$\dot{k}(t) = (r-\pi-g)\,k(t) + w(t)\,n - c(t). \tag{3.24}$$

In the remainder of the chapter, we consider two financing scenarios.

- *Transfer scenario*. Government consumption is zero, and the entire tax revenue is transferred to households, i.e. $g = 0$ and $TR(v,t) > 0$ for all v and t. Within this scenario we consider three prototypical modes of transfer redistribution. To capture these three options we assume that government transfers are set according to:

$$\frac{TR(v,t)}{w(t)} = z\cdot e^{\phi(t-v)}, \tag{3.25}$$

Table 3.1. The model

(a) Microeconomic relationships:

$$\begin{aligned}
\frac{(1+\theta_C)\,C(v,v)}{w(v)} &= \frac{(1-\theta_L)\,\varepsilon_C\left[\alpha_0\Xi(r+\zeta_0-\gamma,\lambda)_S^R-\alpha_1\Xi(r+\zeta_1-\gamma,\lambda)_S^R\right]}{(1-\varepsilon_C)\,\Xi(\rho,1)_S^R+\varepsilon_C\Xi(\rho,1)_0^{\bar{D}}} \\
&\quad+\frac{\varepsilon_C\Xi(z,r-\phi-\gamma,\lambda)_0^{\bar{D}}\cdot z}{(1-\varepsilon_C)\,\Xi(\rho,1)_S^R+\varepsilon_C\Xi(\rho,1)_0^{\bar{D}}} &\text{(T1.1)}\\
\frac{(1+\theta_C)\,C(v,v)}{w(v)} &= \frac{\varepsilon_C}{1-\varepsilon_C}E(S)\,(1-\theta_L)\,e^{-(r-\gamma-\rho)S+(1-\lambda)M(S)} &\text{(T1.2)}\\
\frac{(1+\theta_C)\,C(v,v)}{w(v)} &= \frac{\varepsilon_C}{1-\varepsilon_C}E(R)\,(1-\theta_L)\,e^{-(r-\gamma-\rho)R+(1-\lambda)M(R)} &\text{(T1.3)}\\
\frac{A(v,v+u)}{w(v)}\cdot e^{-ru-\lambda M(u)} &= -\frac{(1+\theta_C)\,C(v,v)}{w(v)}\cdot\Xi(\rho,1)_0^u+z\cdot\Xi(r-\phi-\gamma,\lambda)_0^u &\text{(T1.4a)}\\
&= -\frac{(1+\theta_C)\,C(v,v)}{w(v)}\cdot\left[\Xi(\rho,1)_0^S+\frac{1}{\varepsilon_C}\Xi(\rho,1)_S^u\right]+z\cdot\Xi(r-\phi-\gamma,\lambda)_0^u \\
&\quad+(1-\theta_L)\left[\alpha_0\Xi(r+\zeta_0-\gamma,\lambda)_S^u-\alpha_1\Xi(r+\zeta_1-\gamma,\lambda)_S^u\right] &\text{(T1.4b)}\\
&= \frac{(1+\theta_C)\,C(v,v)}{w(v)}\cdot\Xi(\rho,1)_u^{\bar{D}}-z\cdot\Xi(r-\phi-\gamma,\lambda)_u^{\bar{D}} &\text{(T1.4c)}
\end{aligned}$$

(b) Macroeconomic relationships:

$$\begin{aligned}
0 &= g\cdot k(t)+z\cdot w(t)\,\beta\Xi(\pi-\phi,1)_0^{\bar{D}}-\theta_C c(t)-\theta_L w(t)\,n \\
&\quad-(1-\lambda)\cdot w(t)\int_0^{\bar{D}}\beta e^{-(\pi+\gamma)u-M(u)}\mu(u)\,\frac{A(v,v+u)}{w(v)}du &\text{(T1.5)}\\
\gamma &\equiv \frac{\dot{k}(t)}{k(t)}=r-\pi-g+\left[n-\frac{c(t)}{w(t)}\right]\cdot\frac{w(t)}{k(t)} &\text{(T1.6)}\\
\frac{w(t)\,n}{k(t)} &= (1-\varepsilon_K)\,\Omega_0 &\text{(T1.7)}\\
n &= \beta\cdot\Bigg[\alpha_0\Xi(\pi+\beta_0,1)_S^R-\alpha_1\Xi(\pi+\beta_1,1)_S^R \\
&\quad-\frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v)}{w(v)}\frac{1+\theta_C}{1-\theta_L}\cdot\Xi(\pi+\rho+\gamma-r,2-\lambda)_S^R\Bigg] &\text{(T1.8)}\\
\frac{c(t)}{w(t)} &\equiv \frac{C(v,v)}{w(v)}\cdot\beta\Xi(\pi+\rho+\gamma-r,2-\lambda)_0^{\bar{D}} &\text{(T1.9)}
\end{aligned}$$

Note: The expressions (T1.4a)–(T1.4c) are valid for, respectively, $0\le u\le S$, $S\le u\le R$, and $R\le u\le\bar{D}$. Either g or z balances the government budget.

where z is the policy tool assuring budget balance and ϕ is the parameter governing the government's choice of redistribution scheme. The government either gives the same lump-sum transfer to everyone (captured by setting $\phi = 0$), redistributes with a bias toward the young ($\phi = -1/\bar{D}$), or redistributes with a bias toward the elderly ($\phi = +1/\bar{D}$). The equilibrium value for z follows from the government budget constraint:

$$\begin{aligned} & z \cdot w(t) \cdot \beta\Xi(\pi - \phi, 1)_0^{\bar{D}} \\ = \quad & \theta_C c(t) + \theta_L w(t) n + (1 - \lambda) \int_{t-\bar{D}}^{t} \mu(t - v) p(v, t) A(v, t) \, dv. \end{aligned}$$

- *Wasteful scenario.* Government tax revenue is entirely spent on wasteful government consumption and transfers are zero, i.e. $g > 0$ and $TR(v, t) = 0$ for all v and t. The spending parameter g is endogenously determined by the government budget constraint:

$$g \cdot k(t) = \theta_C c(t) + \theta_L w(t) n + (1 - \lambda) \int_{t-\bar{D}}^{t} \mu(t - v) p(v, t) A(v, t) \, dv.$$

3.2.3 Balanced growth path

We need to tidy up some loose ends. We remember that output and the factor prices can be written as:

$$y(t) = \Omega_0 k(t), \qquad w(t) n = (1 - \varepsilon_K) y(t), \tag{3.26}$$

where $k(t) \equiv K(t) / P(t)$ is the per capita stock of capital and $y(t) \equiv Y(t) / P(t)$ is per capita output. The macroeconomic growth model has been written in a compact format in Table 1. In various places the demographic function (3.14) has been applied. Equation (T1.1) is obtained by using (3.11)–(3.12), (3.4), and (3.25). Equations (T1.2)–(T1.3) follow directly from (3.9)–(3.10). The scaled asset path, (T1.4), has been derived by solving (3.2) and features three segments, depending on the agent's life-cycle phase. Equation (T1.5) is the government budget equation in its most general form. Equation (T1.6) is a slightly rewritten version of (3.24), whilst (T1.7) follows from the expressions in (3.26). Equation (T1.8) is obtained by using (3.4) in (3.19). Finally, equation (T1.9) is the same as (3.18).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Equations (T1.1)–(T1.4) determine scaled newborn consumption, $C(v,v)/w(v)$, the optimal labour market entry and retirement ages, S and R, and the life-cycle path for assets as a function of the key macroeconomic variables (γ and z) and the tax rates (θ_C and θ_L). In turn, equations (T1.5)–(T1.9) determine equilibrium transfers or wasteful spending, z or g, the macroeconomic growth rate, γ, the overall wage-capital ratio, $w(t)/k(t)$, aggregate labour supply in efficiency units, n, and the $c(t)/w(t)$ ratio as a function of the microeconomic variables.

We parameterize the model to capture the key features of an advanced economy. The productivity and demographic parameters[1] are taken from the estimates outlined in Chapter 2. For the set of structural parameters we assume that $r = 0.06$, $\rho = 0.035$, $\pi = 0.01$, and $\delta = 0.10$. Using π in combination with the demographic parameters in (3.15) we find that $\beta = 0.0234$. The remaining parameters are used for calibration purposes. The utility parameter, ε_C, has been chosen so as to induce retirement at model age $u = 42$ (i.e., 60 years in biological age), the capital share, ε_K, has been chosen to assure a growth rate of two percent per annum in the initial calibration, and Ω_0 is set to produce the right interest rate. We find $\Omega_0 = 0.6661$, $\varepsilon_K = 0.2402$, and $\varepsilon_C = 0.0935$. The policy parameters $\{\theta_C, \theta_L, \phi, z, g\}$ and the wedge factor, λ, are set according to need in the separate calibrations below. In the initial calibration annuities are perfect, i.e. $\lambda = 1$. For an imperfect annuity market we assume that $\lambda = 0.7$.[2]

The main features of the benchmark calibration are reported in column (a) of Table 2.[3] Figure 3 shows some life-cycle features of the initial model calibration with lump-sum transfers but without consumption and labour income taxes. The solid line indicates the scenario with a perfect annuity market ($\lambda = 1$), whereas the dotted line indicates the scenario with an imperfect annuity market ($\lambda = 0.7$). Figure 3(a) shows

[1] Remember that we consider agents from age 18 onward so that a model age of 0 corresponds to a biological age of 18.

[2] These values relate to the literature as follows. Auerbach and Kotlikoff (1987, pp. 35, 50-53) assume that $r = 0.067$, $\pi = 0.01$, and $\varepsilon_K = 0.25$. They assume that the intratemporal substitution elasticity between consumption and leisure is 0.8, which is close to the unitary value used by us. The depreciation rate, δ, is in the range reported in the empirical literature (e.g., Nadiri and Prucha, 1996, p. 49). For the annuity wedge, $1 - \lambda$, we use the values inferred from Friedman and Warshawsky (1988) and Hansen and İmrohoroğlu (2008).

[3] Naturally, columns (a) and (b) of Table 2 correspond to column (d) in Table 2 of Chapter 2.

that individuals enter the labour market at age 7.47, quickly increase labour supply to full-time levels, about twenty percent of the total time endowment, and then smoothly ease into retirement at age 42. Figure 3(b) shows that, in the presence of perfect annuity markets, individuals opt for an ever increasing consumption level, whereas in the presence of imperfect annuity markets, they choose a hump-shaped consumption profile. The latter effect is a direct consequence of equation (3.13) where we see that $\lambda < 1$ causes individuals to discount future consumption by a term proportional to their instantaneous probability of death, $\mu\,(t-v)$. As $\mu\,(t-v)$ is increasing in age, future consumption is discounted more heavily. In terms of assets, the hump-shaped consumption profile implies a lower demand for assets to finance old-age consumption. Hence, the imperfect (dotted) annuity path lies below the perfect (solid) annuity path in Figure 3(c). The decrease in capital accumulation translates into lower growth (see the column (b) in Table 2) because capital accumulation – including the externalities associated with it – constitutes the engine of endogenous growth. Finally, Figure 3(d) shows that the individual's scaled wage path is monotonically increasing over the life-cycle. Hence, the macroeconomic growth effect dominates the reduction in labour productivity for ageing workers.

From the initial calibration in Figure 3 we see that the core model with imperfect annuities captures the basic features of the empirically observed life-cycle. In particular, individuals exhibit a hump-shaped profile in consumption (although peaking somewhat late in life), labour supply, and assets. Furthermore, from a macroeconomic perspective, we see that the model captures the broad features of the aggregate economy. That is, the model exhibits an economic growth rate of 2% and implies a capital efficiency parameter of 0.24. In combination, the realistic life-cycle and macroeconomic features allow us to analyze how public policy influences the intergenerational allocation of resources and affects economic growth. In the following sections we study two classical public policies, namely consumption and labour income taxation.

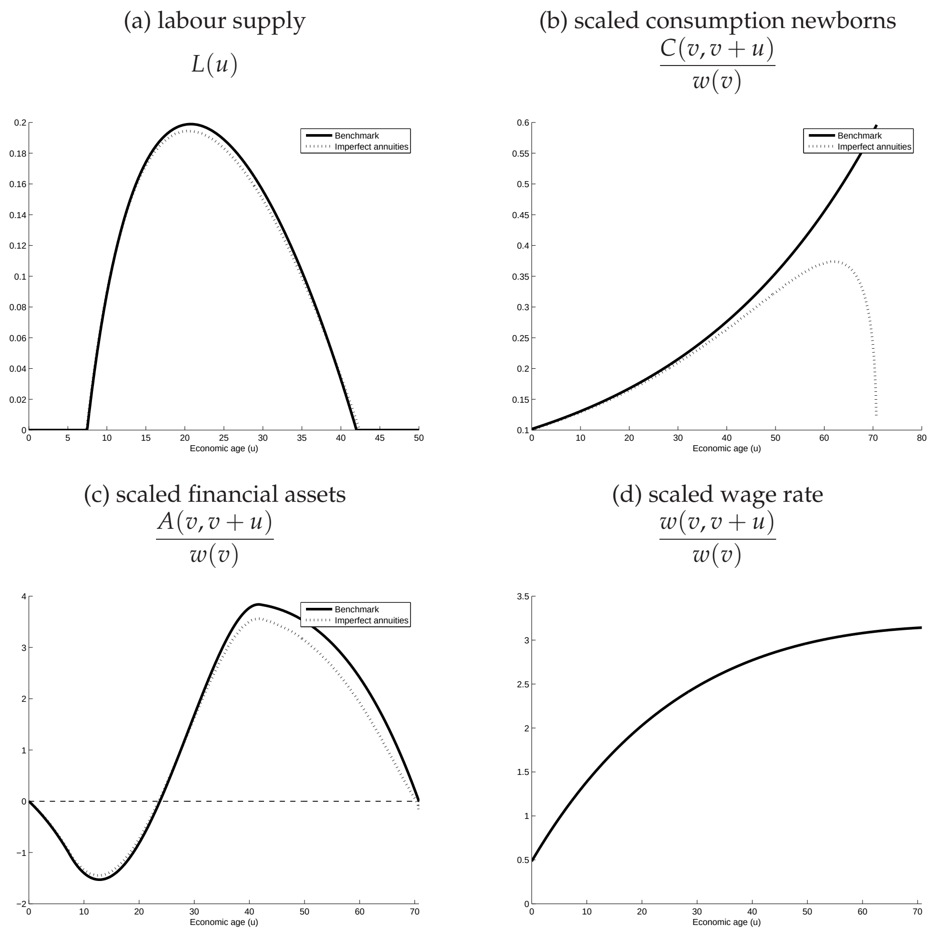
3.3 Consumption tax

Figure 3 shows the effects of a twenty percent tax on consumption expenditures ($\theta_C = 0.2$). The labour income tax is assumed to be zero ($\theta_L = 0$). The solid line is the

Table 3.2. Taxation, retirement, and growth: quantitative effects

	Benchmark		*Consumption Tax*		*Labour Tax*		
	(a) $\lambda = 1.0$	(b) $\lambda = 0.7$	(c) $\lambda = 1.0$	(d) $\lambda = 0.7$	(e) $\lambda = 1.0$	(f) $\lambda = 0.7$	(g) $\lambda = 1.0$ $\theta_L = 0.225$
$\frac{C(v,v)}{w(v)}$	0.1017	0.1007	0.0878	0.0867	0.0648	0.0638	0.0841
S (years)	7.47	7.40	8.22	8.11	9.00	8.81	8.26
R (years)	42.00	42.40	41.05	41.27	38.15	38.01	40.00
z	0	0.0025	0.0199	0.0216	0.0281	0.0285	0.0199
γ (%)	2	1.89	2.05	1.93	1.99	1.86	2
n	0.1101	0.1079	0.0939	0.0920	0.0703	0.0688	0.0882
$\frac{c(t)}{w(t)}$	0.1167	0.1145	0.0994	0.0976	0.075	0.073	0.0934
$\frac{w(t)}{k(t)}$	4.60	4.70	5.39	5.50	7.20	7.36	5.74
$\mathbb{E}\Lambda(v_0, v_0)$	−6.9474	−7.2824	−7.0474	−7.4007	−7.5224	−7.8984	−7.1808

Figure 3.2. General equilibrium in the core model



benchmark calibration (featuring $\theta_C = \theta_L = 0$), the dashed line is the benchmark calibration with the consumption tax imposed, and the thin dotted line is the model with the consumption tax and imperfect annuities. Figure 3(a) reveals that the consumption tax prompts individuals to enter the labour market later, to spend fewer hours working and to retire earlier. See columns (a) and (c) in Table 2 for the quantitative effects. Figure 3(b) shows that consumption decreases at all ages in the presence of taxes. Intuitively, the consumption tax increases the retail price of consumption goods. Furthermore, the consumption tax acts as an implicit labour tax, hence individuals decrease their supply of labour. As before, imperfections on the annuity market lead to a hump-shaped profile in consumption.

In terms of growth we see in column (c) of Table 2 that steady-state growth is slightly *higher* in the presence of consumption taxes. As can be seen in Figure 3(d) this is a direct consequence of intergenerational redistribution effects arising from the recycling of tax revenues. Indeed, scaled *net* transfers to individuals are defined as follows:

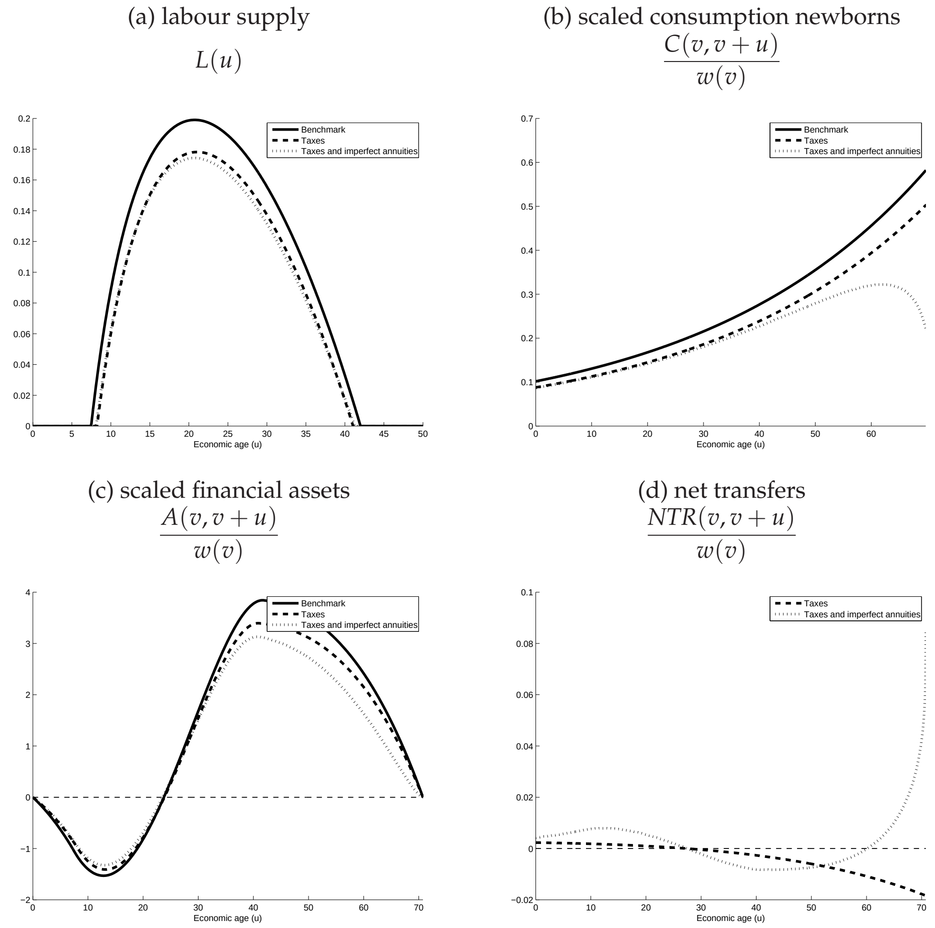
$$\frac{NTR\,(v,t)}{w\,(v)} \equiv \frac{TR\,(v,t)}{w\,(v)} - \theta_L \frac{w(v,t)}{w\,(v)} L\,(v,t) - \theta_C \frac{C(v,t)}{w\,(v)} - (1-\lambda)\,\mu\,(t-v)\,\frac{A\,(v,t)}{w\,(v)}, \tag{3.27}$$

where the first element of right-hand side are the transfers received from the government, the second and the third elements are the taxes paid, and the final term represents the part of annuity income that is lost due to the imperfection on the annuity market. Over the life-cycle an individual may alternate between being a net recipient of transfers and a net donor of transfers because the four elements in equation (3.27) exhibit different life-cycle patterns – see Figure 3.

In Figure 3(d) we see that under consumption taxation, individuals are net recipients of transfers when young whilst they are net donors when old. Figure 4(d) shows that this result follows from the rising profile of consumption. As the young are accumulators of assets, and the elderly are decumulators of assets this leads to an increase in aggregate capital accumulation, and, hence, an increase in economic growth.

If annuity market imperfections are also taken into account, we find that individuals start out as net recipients, are donors during mid-life, and become recipients again later in life. Again from Figure 3(d) we see that part of the different transfer profile is

Figure 3.3. Consumption Taxation



due to the hump-shape in consumption induced by the annuity market imperfection. Furthermore, from Figure 3(c) we see that the implicit tax on annuity income falls especially on the wealthy middle-aged so that the elderly and the young benefit from the transfers whereas the middle aged pay. The growth effect, however, remains positive as a comparison between columns (b) and (d) in Table 2 reveals.

The positive growth effect, however, does not carry over to welfare. Comparing columns (a) and (c) (or (b) and (d)) in Table 2 reveals that individual welfare, measured as the discounted utility of the steady-state generation, decreases in the presence of consumption taxation. From Figures 3(a) and (b) we observe that there are two opposing forces at work. On the one hand, welfare increases because agents supply less labour over the life-cycle, see also the n row in Table 2. On the other hand, life-time consumption decreases. At the aggregate level the consumption effect outpaces the leisure effect so that welfare decreases.

To highlight the impact of intergenerational redistribution, we study the consequences of alternative redistribution schemes in Figure 4. The solid line assumes that the transfers are spread evenly ($\phi = 0$, the case discussed above), in the dashed line we skew the distribution of transfers toward the elderly ($\phi = +1/\bar{D}$) and in the dotted line we skew the transfers toward the young ($\phi = -1/\bar{D}$). In Table 3 these cases corresponds to the second, third, and fourth column, respectively. Comparing rows (a) and (c) (or indeed (b) and (d)) we find that the positive impact on growth disappears for the regime in which transfers are skewed toward the elderly. In contrast, the positive impact is enhanced if transfers are skewed toward the young. This observation allows us to conclude that the increase in the growth rate arises from the intergenerational redistribution effect that channels funds from the decumulating elderly to the accumulating young.

A similar effect is found in Heijdra and Ligthart (2000, p. 697) where it is shown that a consumption tax may lead to a higher steady-state capital stock; the exogenous growth equivalent of a higher growth rate. However, Heijdra and Ligthart dismiss this possibility as empirically unlikely because it would require an unreasonably high instantaneous probability of death. The discrepancy between their results and ours arises from the fact that they assume a constant instantaneous probability of death whereas we assume an age-dependent one.

Figure 3.4. Alternative transfers

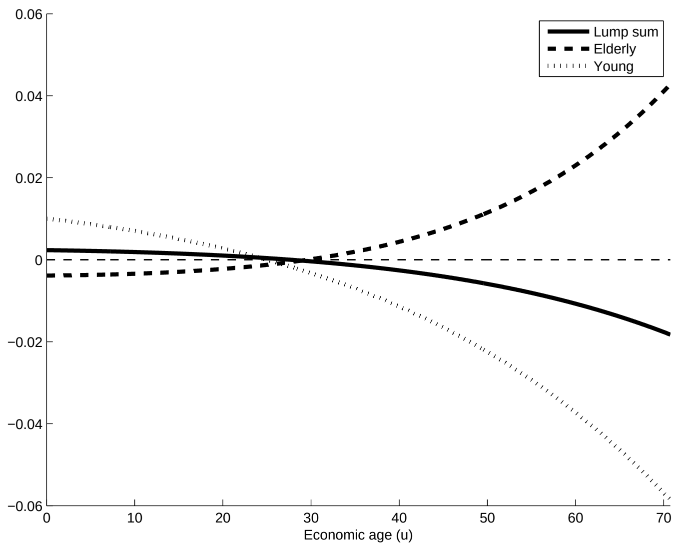


Table 3.3. Economic growth*

	$\{\lambda, \theta_C, \theta_L\}$	$\phi = 0$	$\phi = +\frac{1}{\bar{D}}$	$\phi = -\frac{1}{\bar{D}}$	$z = 0$
(a)	$\{1, 0, 0\}$	2.00	2.00	2.00	2.00
(b)	$\{0.7, 0, 0\}$	1.89	1.88	1.90	1.69
(c)	$\{1, 0.2, 0\}$	2.05	1.95	2.14	−1.20
(d)	$\{0.7, 0.2, 0\}$	1.93	1.81	2.03	−1.53
(e)	$\{1, 0, 0.4\}$	1.99	1.72	2.24	−2.60
(f)	$\{0.7, 0, 0.4\}$	1.86	1.57	2.13	−2.96

*Cell entries show the percentage of economic growth per annum in the respective scenario.

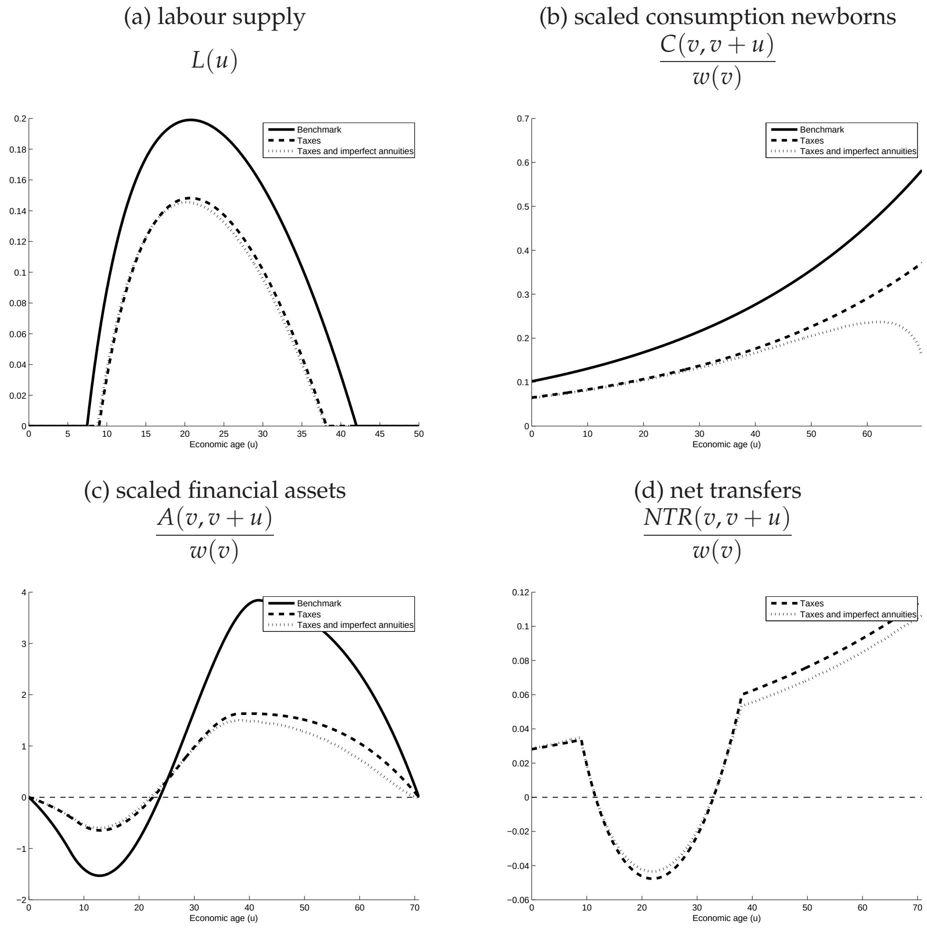
To emphasize the importance of government transfers in general, we conclude this section with a study of the case in which the government pursues the wasteful scenario, i.e. tax revenue is spent entirely on wasteful government expenditure (see above). The growth effects are reported in the final column of Table 3. The comparison of row (a) and (c) (or (b) and (d)) immediately reveals the detrimental growth effects of wasteful government expenditures. For consumption taxation and annuity market imperfection the growth rate drops from 2 percent in the initial calibration to -1.20 percent and -1.53 percent for the pure taxation and the combined case, respectively. These detrimental effects derive from the fact that the government now not only distorts the market through taxation but also drains resources from the economy for wasteful purposes.

3.4 Labour income tax

In Figure 6 we study the impact of a forty percent tax on labour income ($\theta_L = 0.4$). The consumption tax is assumed to be zero ($\theta_C = 0$). As before, the solid lines represent the benchmark case, the dashed line the scenario with the labour income tax alone, and the dotted line the taxes and imperfect annuities case. Figure 6(a) shows that the labour tax induces individuals to postpone labour market entry, to work less during their active career and to retire early. Compare the columns (a) and (e) in Table 2 for the quantitative effects. In Figure 5(b) we see that the tax decreases initial consumption and that the annuity market imperfection causes the familiar hump-shaped profile. Finally, Figure 5(c) reveals that agents accumulate less debt early on in life but also substantially fewer assets later on in life.

From an intuitive point of view the labour tax decreases the benefits from work, so that individuals reduce their labour effort, both on the intensive and the extensive margin. The labour tax also acts as an implicit tax on consumption, hence consumption is reduced in a fashion akin to the imposition of the explicit consumption tax studied above. Even though the consumption and labour tax act as each others implicit equivalents, the standard labour-consumption tax equivalence result (Atkinson and Stiglitz, 1980) no longer holds. The failure of the equivalence results is also derived in Heijdra and Ligthart (2000) where it is shown that the life-cycle paths of consump-

Figure 3.5. Labour Taxation



tion and labour induced by finite lives lead to asymmetric tax incidence of labour and consumption. In our model the equivalence result also fails because individuals retire.

In terms of growth we find an asymmetry between consumption and labour taxes, cf. columns (c) and (e) in Table 2. Whereas consumption taxes lead to a slight *increase* in growth, labour taxes lead to a slight *decrease* in growth. From Figure 5(d) we see that this is a consequence of the different intergenerational redistribution structure. Where the consumption tax induced redistribution from the decumulating elderly to the accumulating young, the labour tax induces redistribution from the working to the idle. Because idleness is an attribute of the elderly, the labour tax causes a redistribution from the accumulating workers to the decumulating retired, hence depressing growth.

In terms of welfare we find a much more pronounced effect for labour-income taxation than for consumption taxation, that is, whereas the welfare effect of a consumption tax was relatively mild, the welfare effect of a labour-income tax is substantial. Comparing Figures 5(a) and (b) reveals once more that the positive welfare effect resulting from a decrease in labour supply is outpaced by the negative effect resulting from the drop in consumption.

The comparative effects of consumption and labour-income taxation on growth and welfare suggest that switching from labour taxation to consumption taxation may be growth as well as welfare enhancing. Indeed, if we study the impact of a labour tax that is revenue neutral with respect to the consumption tax (compare columns (c) and (g)) we find that both in terms of growth and welfare the consumption tax dominates the labour tax.

In rows (e)–(f) of Table 3 we study the consequences of alternative redistributions schemes once more. In accordance with the variational exercise depicted in Figure 5, we find that a redistribution scheme skewed toward the elderly has detrimental effects on economic growth, whereas a redistribution scheme skewed toward the young has advantageous effects on economic growth. Notice especially, that the positive growth effect of a redistribution skewed toward the young outpaces the positive effect found under consumption taxes. From the entries for z in Table 2 we see that is because the pie to be redistributed is larger under labour taxation.

Finally, in rows (e)–(f) of the last column of Table 3 we repeat the analysis of a wasteful government for the case of labour income taxes. As before we find that wasteful government expenditure causes huge detrimental effects on economic growth. In particular, for labour income taxes and annuity market imperfection the growth rate drops from 2% to −2.60% and −2.96% for the pure taxation and combined case, respectively. In accordance with the consumption taxation case the detrimental effect is driven by the government's drain of productive resources. The absolute magnitude of the effect is bigger than in the consumption taxation case because government revenue derived from labour taxation is larger.

3.5 Conclusions

In this chapter we have studied two classical themes in dynamic public finance theory; consumption and labour income taxation. In the analysis we have paid special attention to alternative modes of redistributing the proceeds of taxation. That is we have analyzed three systems that either redistribute government revenue evenly across all individuals or with a bias toward the young or the old. Finally, to emphasize the importance of redistribution of government income in general we have also analyzed a wasteful government that uses the proceeds from taxation for useless consumptive purposes.

We have found that both consumption and labour income taxation lead to an increase in economic growth if the proceeds are redistributed with a bias toward the young. This is due to the beneficial intergenerational redistribution effects that channel resources from the dissaving elderly to the saving young. In addition, we have found that the two taxes lead to lower growth if proceeds are redistributed toward the elderly. In this case, the intergenerational redistribution effect is negative because resources flow from the saving young to the dissaving elderly. For the case of lump-sum transfers we have found that growth increases slightly for consumption taxation and decreases slightly for labour income taxation. The differing consequences of the two taxes are, again, due to intergenerational redistribution effects. The consumption tax redistributes funds from the elderly, who are strong consumers and thus pay the lion's share of tax, to the young, who barely consume but save a lot. The labour income tax, on the other hand, redistributes funds between the working and the idle. Idleness being an attribute of the retired, the tax induces redistribution from saving workers to consuming retirees. Finally, we have found that wasteful government expenditures have strong detrimental effects on growth. Besides distorting the economy through the taxation, the government also drains productive resources.

Having studied various aspects of labour-income and consumption taxation we now turn our attention to the role of the pension system. That is, in the next chapter we modify the model from Chapter 2 to study how various public pension designs moderate the impact of a demographic shock.

Chapter 4

Pensions and ageing*

* This chapter is based on Heijdra and Mierau (2011).

4.1 Introduction

The coming generational storm is arguably the strongest tempest against which current and future politicians have to sail. The policy-panacea, however, has not yet been developed and, maybe even worse, there is no consensus on what the economic consequences of an ageing population actually are. Hence, in this chapter we use a simplified version of the model developed in Chapter 2 to study the macroeconomic consequences of an increase in old age mortality. We introduce a simple Pay-As-You-Go (PAYG) pension system, which may either be run on a defined benefit (DB) or a defined contribution (DC) basis. In the former case the replacement rate acts as a policy variable whereas in the latter case the contribution rate is the policy variable. Furthermore, and in contrast to the previous chapters, we let retirement be exogenous, which allows us to consider the retirement age as a policy variable.

We find that, in principle, ageing is good for economic growth because it increases the incentive for individuals to save. However, if a defined benefit system is in place the higher contributions necessary to finance the entitlement of the additional pensioners will reduce individual savings and thereby dampen the growth increase following a longevity shock. In order to circumvent this reduction in growth the government could opt to introduce a defined contribution system in which the benefits are adjusted downward to accommodate the increased dependency ratio. Surprisingly, we find that if the government increases the retirement age such that the old age dependency ratio remains constant economic growth drops compared to both the defined benefit and the defined contribution system. This is due to an adverse savings effect following from the shortened retirement period.

The remainder of this chapter is set-up as follows. The next section introduces the model and discusses how we feed in a realistic life-cycle. Section 3 analyses the steady-state consequences of ageing and provides some policy recommendations. The final section concludes.

4.2 Model

We use the model developed in Chapter 2 and extend it to include a public pension system on the government side but simplify it by making the retirement decision exogenous on the consumer side. We refer the reader to Chapter 2 for a complete description of the model and use the remainder of this section to outline the model and its the extensions.

4.2.1 Consumers

Individual behaviour

We develop the individual's decision rules from the perspective of birth. Expected lifetime utility of an individual born at time v is given by:

$$\mathbb{E}\Lambda\left(v,v\right) \equiv \int_{v}^{v+\bar{D}} \frac{C(v,\tau)^{1-1/\sigma}-1}{1-1/\sigma} \cdot e^{-\rho(\tau-v)-M(\tau-v)} d\tau, \tag{4.1}$$

where $C\left(v,\tau\right)$ is consumption, σ is the intertemporal substitution elasticity ($\sigma > 0$), ρ is the pure rate of time preference ($\rho > 0$), $\bar{D}$ is the maximum attainable age for the agent, and $e^{-M(\tau-v)}$ is the probability that the agent is still alive at some future time τ ($\geq v$). Here, $M(\tau-v) \equiv \int_0^{\tau-v} \mu(s)ds$ stands for the cumulative mortality rate and $\mu\left(s\right)$ is the instantaneous mortality rate of an agent of age s.

The agent's budget identity is given by:

$$\dot{A}(v,\tau) = r^{A}\left(\tau-v\right)A(v,\tau) + w(v,\tau)L\left(v,\tau\right) - C(v,\tau) + PR\left(v,\tau\right) + TR\left(v,\tau\right), \tag{4.2}$$

where $A\left(v,\tau\right)$ is the stock of financial assets, $r^{A}\left(\tau-v\right)$ is the age-dependent annuity rate of interest rate, $w\left(v,\tau\right) \equiv E\left(\tau-v\right)w\left(\tau\right)$ is the age-dependent wage rate, $E\left(\tau-v\right)$ is exogenous labour productivity, $L\left(v,\tau\right)$ is labour supply, $PR(v,\tau)$ are payments received from the public pension system, and $TR\left(v,\tau\right)$ are lump-sum transfers (see below). Labour supply is exogenous and mandatory retirement takes place at age

R. Since the time endowment is unity, we thus find:

$$L(v,\tau) = \begin{cases} 1 & \text{for } 0 \le \tau - v < R \\ 0 & \text{for } R \le \tau - v < \bar{D} \end{cases}. \tag{4.3}$$

There is a simple PAYG pension system which taxes workers and provides benefits to retirees:

$$PR(\tau) = \begin{cases} -\theta w(v,\tau) & \text{for } 0 \le \tau - v < R \\ \zeta w(\tau) & \text{for } R \le \tau - v < \bar{D} \end{cases} \tag{4.4}$$

where $w(\tau)$ is the economy wide wage rate, θ $(0 < \theta < 1)$ is the contribution rate and ζ is the benefit rate $(\zeta > 0)$. Under a DC system, θ is exogenous and ζ adjusts to balance the budget (see below). The opposite holds under a DB system. Finally, we postulate that lump-sum transfers are age-independent:

$$TR(v,\tau) = z \cdot w(\tau), \tag{4.5}$$

where z is endogenously determined via the balanced budget requirement of the redistribution scheme (see below).

In order to replicate the salient features of the individual life-cycle we add a number of distinguishing features to our model. As these features have been discussed at length in Chapters 2-3 we suffice by stating them here and referring the reader to the previous chapters.

Imperfect annuity markets: The annuity rate of interest facing the agent is given by:

$$r^A(\tau - v) \equiv r + \lambda\mu(\tau - v), \qquad (\text{for } 0 \le \tau - v < \bar{D}). \tag{4.6}$$

where r is the real interest rate and λ is a parameter $(0 < \lambda \le 1)$ indicating the degree of imperfection on the annuity market.

Age-dependent productivity: Labour productivity is hump-shaped over the life-cycle. A useful parameterization of the productivity profile is:

$$E(t - v) = \alpha_0 e^{-\zeta_0(t-v)} - \alpha_1 e^{-\zeta_1(t-v)}, \qquad (\text{for } 0 \le t - v \le \bar{D}), \tag{4.7}$$

where α_i and ζ_i are the parameters governing the curvature of the productivity profile (see Box 2.1 for details).

Age-dependent mortality: Individual mortality increases over the life-cycle. We assume that $e^{-M(t-v)}$ takes the following useful functional form:

$$e^{-M(t-v)} \equiv \frac{\eta_0 - e^{\eta_1 (t-v)}}{\eta_0 - 1}, \qquad (\text{for } 0 \leq t-v \leq \bar{D}), \tag{4.8}$$

where $\eta_0 > 1$ and $\eta_1 > 0$ are parameters (see Box 2.2 for details).

The agent chooses time profiles for $C(v,\tau)$ and $A(v,\tau)$ (for $v \leq \tau \leq v+\bar{D}$) in order to maximize (4.1), subject to (i) the budget identity (4.2), (ii) a transversality condition, $A(v, v+\bar{D}) = 0$, and (iii) the initial asset position at birth, $A(v,v) = 0$. The optimal consumption profile for a vintage-v individual of age u ($0 \leq u \leq \bar{D}$) is fully characterized by the following equations:

$$\begin{aligned}
C(v, v+u) &= C(v,v) \cdot e^{\sigma[(r-\rho)u-(1-\lambda)M(u)]}, && (4.9)\\
\frac{C(v,v)}{w(v)} &= \frac{1}{\int_0^{\bar{D}} e^{(\sigma-1)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]} ds} \cdot \frac{H(v,v)}{w(v)}, && (4.10)\\
\frac{H(v,v)}{w(v)} &= (1-\theta)\int_0^R E(s)\, e^{-(r-g)s-\lambda M(s)} ds + \zeta \int_R^{\bar{D}} e^{-(r-g)s-\lambda M(s)} ds \\
&\quad + z \int_0^{\bar{D}} e^{-(r-g)s-\lambda M(s)} ds. && (4.11)
\end{aligned}$$

The intuition behind these expressions is as follows. Equation (4.9) is best understood by noting that the consumption Euler equation resulting from utility maximization takes the following form:

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = \sigma \cdot [r - \rho - (1-\lambda)\,\mu(\tau - v)]. \tag{4.12}$$

By using this expression, future consumption can be expressed in terms of consumption at birth as in (4.9). In the absence of an annuity market imperfection ($\lambda = 1$), consumption growth only depends on the gap between the interest rate and the pure rate of time preference. In contrast, with imperfect annuities, individual consumption growth is negatively affected by the mortality rate, a result first demonstrated for the case with $\lambda = 0$ by Yaari (1965, p. 143).

Equation (4.10) shows that scaled consumption of a newborn is proportional to scaled human wealth. Finally, equation (4.11) provides the definition of human wealth at birth. The first term on the right-hand side represents the present value of the time endowment during working life, using the growth-corrected annuity rate of interest for discounting. The second term on the right-hand side denotes the present value of the pension received during retirement. Finally, the third term on the right-hand side of (4.11) is just the present value of transfers arising from the annuity market imperfection.

The asset profiles accompanying the optimal consumption plans are given for a working-age individual ($0 \leq u < R$) by:

$$\begin{aligned}\frac{A\left(v,v+u\right)}{w\left(v\right)}e^{-ru-\lambda M(u)} &= (1-\theta)\int_0^u E\left(s\right)e^{-(r-g)s-\lambda M(s)}ds + z\int_0^u e^{-(r-g)s-\lambda M(s)}ds \\ &\quad -\frac{C(v,v)}{w\left(v\right)}\int_0^u e^{(\sigma-1)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds, \qquad (4.13)\end{aligned}$$

and for a retiree ($R \leq u \leq \bar{D}$) by:

$$\begin{aligned}\frac{A\left(v,v+u\right)}{w\left(v\right)}e^{-ru-\lambda M(u)} &= \frac{C(v,v)}{w\left(v\right)}\int_u^{\bar{D}} e^{(\sigma-1)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds \\ &\quad -\left(\zeta+z\right)\int_u^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds. \qquad (4.14)\end{aligned}$$

Aggregate household behaviour

In general, we can define per-capita average values in general terms as:

$$x\left(t\right) \equiv \int_{t-\bar{D}}^{t} p\left(v,t\right)X\left(v,t\right)dv, \qquad (4.15)$$

where $X\left(v,t\right)$ denotes the variable in question at the individual level, $x\left(t\right)$ is the per capita average value of that same variable, and $p\left(v,t\right) \equiv \beta e^{-\pi(t-v)-M(t-v)}$ are the cohort weights (see Box 2.2 for details).

Per capita aggregate household behaviour is summarized by the following expressions:

$$\frac{c\left(t\right)}{w\left(t\right)} = \beta\frac{C(v,v)}{w\left(v\right)}\int_0^{\bar{D}} e^{\sigma[(r-\rho)s-(1-\lambda)M(s)]-(\pi+g)s-M(s)}ds, \qquad (4.16)$$

$$\begin{aligned} n(t) &= n \equiv \beta \int_0^R E(s)\, e^{-\pi s - M(s)} ds, && (4.17) \\ \dot{a}(t) &= (r-\pi)\, a(t) + w(t)\, n(t) - c(t) \\ &\quad + \left[\zeta \int_R^{\bar{D}} \beta e^{-\pi s - M(s)} ds - \theta \int_0^R \beta E(s)\, e^{-\pi s - M(s)} ds \right] w(t) \\ &\quad + \left[(1-\lambda) \int_0^{\bar{D}} \beta e^{-(g+\pi)s - M(s)} \mu(s) \frac{A(v, v+s)}{w(v)} ds - z \right] w(t). && (4.18) \end{aligned}$$

Equation (4.16) relates the macroeconomic consumption-wage ratio to the optimally chosen scaled consumption level by newborns. Since this ratio is time-invariant, per capita consumption grows at the macroeconomic growth rate g. Equation (4.17) shows that aggregate per capita labour supply (in efficiency units) is a time-invariant constant. Finally, the growth rate in per capita financial assets is given in equation (4.18). This expression will be discussed in more detail below.

4.2.2 Loose ends

We assume that the PAYG pension scheme is run on a balanced-budget basis. In view of (4.7), (4.4) and the demographic steady state condition this furnishes the following budget constraint:

$$\zeta w(t) \int_R^{\bar{D}} \beta e^{-\pi s - M(s)} ds = \theta w(t) \int_0^R \beta E(s)\, e^{-\pi s - M(s)} ds, \tag{4.19}$$

where the left-hand side stands for pension payments to retirees and the right-hand side represents pension contributions by workers. The mandatory retirement age R is exogenous. Under the assumption of a DC system, θ is also exogenous and ζ adjusts to balance the budget. The opposite holds under a DB system. In view of (4.19), the PAYG system does not feature in the expression for aggregate asset accumulation, i.e. the second line of (4.18) is zero.

Excess profits of annuity firms can be written as follows:

$$EP(v,t) \equiv (1-\lambda) \int_{t-\bar{D}}^{t} p(v,t)\, \mu(t-v)\, A(v,t)\, dv. \tag{4.20}$$

The integral on the right-hand side represents per capita annuitized assets of all individuals that die in period t. This is the total revenue of annuity firms, of which only a fraction λ is paid out to surviving annuitants. The remaining fraction, $1-\lambda$, is excess profit which is taxed away by the government and disbursed to all households in the form of lump-sum transfers, i.e. $EP(v,t) = TR(v,t)$. Using (4.5) and (4.20) we find the implied expression for z:

$$z = (1-\lambda)\int_0^{\bar{D}} \beta e^{-(g+\pi)u - M(u)} \mu(u) \frac{A(v, v+u)}{w(v)} du. \tag{4.21}$$

Just as for the PAYG system, the redistribution of excess profits of annuity firms also debudgets from the asset accumulation equation, i.e. the third line in (4.18) is also zero.

In the absence of government bonds, the capital market equilibrium condition is given by $A(t) = K(t)$ or, in per capita average terms, by:

$$a(t) = k(t), \tag{4.22}$$

where $k(t) \equiv K(t)/P(t)$ is the per capita stock of capital. As before we easily find:

$$\begin{aligned} y(t) &= \Omega_0 k(t), && (4.23)\\ w(t)\,n(t) &= (1-\varepsilon)\,y(t), && (4.24)\end{aligned}$$

where $y(t) \equiv Y(t)/P(t)$ is per capita output. From (4.18)–(4.19), (4.21) and (4.22) we can derive the expression for the macroeconomic growth rate:

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[n(t) - \frac{c(t)}{w(t)}\right]\frac{w(t)}{k(t)}. \tag{4.25}$$

For convenience, the key equations comprising the general equilibrium model have been gathered in Table 1. Equations (T1.1)–(T1.2), (T1.3a)–(T1.3b), (T1.4)–(T1.6), (T1.8)–(T1.9) restate, respectively, (4.10)–(4.11), (4.13)–(4.14), (4.19), (4.21), (4.25), (4.17), and (4.16). Equation (T1.7) is obtained by combining (4.23) and (4.24) and noting (4.17).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Indeed, conditional on the macroeconomic variables, equations (T1.1)–(T1.3) determine scaled newborn consumption and human wealth, $C(v,v)/w(v)$ and $H(v,v)/w(v)$ as well as the age profile of scaled assets $A(v, v+u)/w(v)$. Conditional on these microeconomic variables, equations (T1.4)–

(T1.9) determine equilibrium pension payments and transfers, ζ and z, the macroeconomic growth rate, g, the overall wage-capital ratio, $w(t)/k(t)$, aggregate labour supply, n, and the $c(t)/w(t)$ ratio.

4.2.3 The core model

For the *core model* we postulate the existence of perfect annuities (PA, with $\lambda = 1$) and parameterized it as follows. The productivity and demographic parameters[1] are taken from the estimates outlined in Chapter 2. We assume that the rate of population growth is half of one percent per annum ($\pi = 0.005$). For the estimated demographic process, the demographic steady-state yields a birth rate equal to $\beta = 0.0204$. Since $\bar{\mu} \equiv \beta - \pi$, this implies that the average mortality rate is $\bar{\mu} = 0.0154$. The old-age dependency ratio equals 22.92%. We model an economy with a steady-state capital-output ratio of 2.5, which is obtained by setting $\Omega_0 = 0.4$. The interest rate is five percent per annum ($r = 0.05$), the capital depreciation rate is seven percent per annum ($\delta = 0.07$), and the efficiency parameter of capital is set at $\varepsilon = 0.3$. The steady-state growth rate is set equal to two percent per annum ($g = 0.02$). For the intertemporal substitution elasticity we use $\sigma = 0.7$, a value consistent with the estimates reported by Attanasio and Weber (1995). The rate of pure time preference is used as a calibration parameter, yielding a value of $\rho = 0.0112$.

Regarding the PAYG pension system we assume that the mandatory retirement age is set at $R = 47$ (corresponding with 65 in biological years) and that the pension contribution rate is seven percent of wage income, i.e. $\theta = 0.07$ which roughly corresponds with the Dutch pension system. The implied pension benefit is determined in general equilibrium.

Table 4.2(a) reports the main features of the initial steady-state growth path. With perfect annuities, there are no excess profits of annuity firms and thus no transfers, i.e. $z = 0$ in Table 4.2(a). Note also that at retirement age R a vintage-v agent receives $\zeta w(v+R)$ in the form of a pension whereas the last-received wage for this agent equals $E(R)\,w(v+R)$. The replacement rate is thus equal to $\zeta/E(R) = 0.3189$.

We visualize the life-cycle profiles for a number a key variables in Figure 1. The

[1] Remember that we consider agents from age 18 onward so that a model age of 0 corresponds to a biological age of 18.

Table 4.1. The model

(a) Microeconomic relationships:

$$\frac{C(v,v)}{w(v)} = \frac{1}{\int_0^{\bar{D}} e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds} \cdot \frac{H(v,v)}{w(v)} \tag{T1.1}$$

$$\frac{H(v,v)}{w(v)} = (1-\theta)\int_0^R E(s)\, e^{-(r-g)s-\lambda M(s)}ds + \zeta \int_R^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds + z\int_0^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds \tag{T1.2}$$

$$\frac{A(v,v+u)}{w(v)} e^{-ru-\lambda M(u)} = (1-\theta)\int_0^u E(s)\, e^{-(r-g)s-\lambda M(s)}ds + z\int_0^u e^{-(r-g)s-\lambda M(s)}ds - \frac{C(v,v)}{w(v)}\int_0^u e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds \tag{T1.3a}$$

$$= \frac{C(v,v)}{w(v)}\int_u^{\bar{D}} e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds - (\zeta+z)\int_u^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds \tag{T1.3b}$$

(b) Macroeconomic relationships:

$$\zeta = \theta \cdot \frac{\int_0^R \beta E(s)\, e^{-\pi s-M(s)}ds}{\int_R^{\bar{D}} \beta e^{-\pi s-M(s)}ds} \tag{T1.4}$$

$$z = (1-\lambda)\int_0^{\bar{D}} \beta e^{-(g+\pi)u-M(u)}\mu(u)\frac{A(v,v+u)}{w(v)}du \tag{T1.5}$$

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r-\pi+\left[n-\frac{c(t)}{w(t)}\right]\frac{w(t)}{k(t)} \tag{T1.6}$$

$$\frac{w(t)\,n}{k(t)} = (1-\varepsilon)\,\Omega_0 \tag{T1.7}$$

$$n = \beta\int_0^R E(s)\, e^{-\pi s-M(s)}ds \tag{T1.8}$$

$$\frac{c(t)}{w(t)} = \beta\frac{C(v,v)}{w(v)}\int_0^{\bar{D}} e^{\sigma[(r-\rho)s-(1-\lambda)M(s)]-(\pi+g)s-M(s)}ds \tag{T1.9}$$

Definitions: Endogenous are $C(v,v)/w(v)$, $H(v,v)/w(v)$, $A(v,v+u)/w(v)$, ζ, z, g, n, $w(t)/k(t)$, and $c(t)/w(t)$. Parameters: R retirement age, θ pension contribution rate, birth rate β, aggregate mortality rate $\bar{\mu}$, population growth rate $\pi \equiv \beta - \bar{\mu}$, imperfection annuities λ, rate of time preference ρ, capital coefficient in the technology ε, scale factor in the technology Ω_0. The interest rate is $r \equiv \varepsilon\Omega_0 - \delta$, where δ is the depreciation rate of capital.

Table 4.2. Quantitative effects

Case:	(a) PA	(b) IA	(c) PA	(d) IA	(e) PA	(f) IA	(g) PA	(h) IA
	Core cases		**DC**		**DB**		**RA**	
$\frac{C(v,v)}{w(v)}$	0.8534	0.8609	1.0784	1.0785	0.9078	0.9053	0.9329	0.9369
$\frac{H(v,v)}{w(v)}$	26.5646	27.0207	36.0229	36.2942	30.3246	30.4653	31.1617	31.5282
g (%)	2.00	1.91	3.36	3.27	2.79	2.68	2.39	2.30
n	0.9675	0.9675	0.8212	0.8212	0.8212	0.8212	0.9589	0.9589
$\frac{w(t)}{k(t)}$	0.2894	0.2894	0.3410	0.3410	0.3410	0.3410	0.2920	0.2920
$\frac{c(t)}{w(t)}$	1.0538	1.0570	0.8692	0.8720	0.8861	0.8893	1.0482	1.0514
ζ	0.3632	0.3632	0.1824	0.1824	0.3632	0.3632	0.3632	0.3632
z		0.0200		0.0142		0.0131		0.0185
θ					0.1394	0.1394		
$R+18$							75.3	75.3

Notes. PA stands for perfect annuities ($\lambda = 1$) and IA denotes imperfect annuities ($\lambda = 0.7$). Column (a) is the core model. Column (b) shows the effects of the annuity market imperfection in the core model. Columns (c)–(d) show the effects of a demographic shock under a DC pension system. Columns (e)–(f) show the effects under a DB system. In this scenario the tax rate θ adjusts to keep ζ at its pre-shock level. Columns (g)–(h) show the effects under a retirement age (RA) scenario in which θ and ζ are kept at their pre-shock levels and R is adjusted.

solid lines are associated with the core model featuring perfect annuities. For ease of interpretation, the horizontal axes report biological age, $u + 18$. Figure 1(a) shows that with perfect longevity insurance consumption rises monotonically over the life cycle. This counterfactual result follows readily from (4.12) which for $\lambda = 1$ simplifies to $\dot{C}(v,\tau)/C(v,\tau) = \sigma(r-\rho)$. Figure 1(b) depicts the age profile of scaled financial assets. At first the agent is a net borrower, i.e. a buyer of life-insured loans. Thereafter annuity purchases are positive and the profile of assets is bell-shaped. In the absence of a bequest motive, the agent plans to run out of financial assets at the maximum age $\bar{D}$. Figure 1(c) shows the profile of scaled wages over the life cycle. Despite the fact that individual labour productivity itself is bell-shaped, wages increase monotonically as a result of ongoing economic growth. Finally, in Figure 1(d) we illustrate the profile for scaled pension receipts. During the working career these payments are negative and proportional to scaled wages, whilst they are positive and proportional to the economy-wide wage rate during retirement.

Despite its simplicity, the model captures some of the main stylized facts regarding life cycles. Indeed, as is documented by *inter alia* Huggett (1996), in real life financial assets typically display a hump-shaped profile and remain non-negative in old age. The model also features a realistic age profile for labour supply. Indeed, as is pointed out by McGrattan and Rogerson (2004) (for the United States), labour supply is constant and age-invariant for most of working life and tapers off rapidly near the retirement age.

In contrast, the model does not provide a realistic profile for consumption. In the core model the age profile for consumption is monotonically increasing, whereas it is hump-shaped in reality. See, for example, Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2007) for evidence on the US, and Alessie and de Ree (2009) for a recent study using Dutch data.

Referring to the consumption Euler equation (4.12) it is clear that an annuity market imperfection can account for a hump-shaped pattern of consumption. Indeed, with $0 < \lambda < 1$ it follows from (4.12) and Figure 1(b) that consumption growth is positive during the early phase of life because the mortality rate is low, i.e. $r - \rho > (1-\lambda)\mu(u)$. Toward the end of life, however, the instantaneous death probability rises sharply, the inequality is reversed, and the optimal consumption profile is downward sloping.[2]

In order to quantify and visualize the effects of an annuity market imperfection we recompute the general equilibrium of the model using the structural parameters mentioned above but setting $\lambda = 0.7$. This degree of annuity market imperfection is in the order of magnitude found by Friedman and Warshawsky (1988, p. 59). Table 2(b) reports the quantitative implications of the annuity market imperfection. Two features stand out. First, in the presence of imperfect annuities excess profits of annuity firms are positive and transfers are thus strictly positive ($z = 0.0200$). Each surviving agent thus receives about two percent of the macroeconomic wage rate in each period in the form of transfers. Second, the macroeconomic growth rate falls by nine basis points, from 2 percent to 1.91 percent per annum.

The ultimate effect on newborn consumption of the change in λ depends on the interplay between the human wealth effect and the propensity effect. Recall from (T1.1)–(T1.2) that $C(v,v) = \Delta \cdot H(v,v)$ where the propensity to consume is defined as:

$$\Delta \equiv \frac{1}{\int_0^{\bar{D}} e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds}. \tag{4.26}$$

It is easy to show that with $0 < \sigma < 1$, the propensity to consume out of human wealth falls as a result of the reduction in λ:

$$\frac{d\Delta}{d\lambda} = (1-\sigma)\,\Delta^2 \cdot \int_0^{\bar{D}} M(s)\, e^{-(1-\sigma)[rs+\lambda M(s)]-\sigma[\rho s+M(s)]}ds > 0. \tag{4.27}$$

The partial derivative of scaled human wealth with respect to λ is given by:

$$\begin{aligned}\frac{\partial}{\partial\lambda}\frac{H(v,v)}{w(v)} &= -(1-\theta)\int_0^{R} M(s)\,E(s)\,e^{-(r-g)s-\lambda M(s)}ds - \zeta\int_R^{\bar{D}} M(s)\,e^{-(r-g)s-\lambda M(s)}ds \\ &\quad -z\int_0^{\bar{D}} M(s)\,e^{-(r-g)s-\lambda M(s)}ds < 0.\end{aligned} \tag{4.28}$$

A decrease in λ results in a reduction in the annuity rate of interest at all age levels and thus an increase in human wealth due to less severe discounting of non-asset in-

[2] Consumption peaks at age $\hat{u}$, which is defined implicitly by $\mu(\hat{u}) = (r-\rho)\,/\,(1-\lambda)$. Since $\mu'(u) > 0$ we find that $d\hat{\mu}/d\lambda > 0$ and $d\hat{\mu}/d\,(r-\rho) > 0$. Hence, the smaller is λ or $r-\rho$, the lower is the age at which consumption peaks. Note that whereas λ can help determine the location of the kink, the intertemporal substitution elasticity σ cannot do so.

come streams. Human wealth is also affected by two of the macroeconomic variables, namely transfers z and the growth rate g (note that n, ζ, and $w(t)/k(t)$ are not affected by λ). Scaled human wealth is boosted as a result of the transfers:

$$\frac{\partial}{\partial z}\frac{H(v,v)}{w(v)} = \int_0^{\bar{D}} e^{-(r-g)s-\lambda M(s)}ds > 0, \tag{4.29}$$

but it is reduced by the decrease in the growth rate:

$$\begin{aligned}\frac{\partial}{\partial g}\frac{H(v,v)}{w(v)} &= (1-\theta)\int_0^R sE(s)\,e^{-(r-g)s-\lambda M(s)}ds + \zeta\int_R^{\bar{D}} se^{-(r-g)s-\lambda M(s)}ds \\ &\quad +z\int_0^{\bar{D}} se^{-(r-g)s-\lambda M(s)}ds > 0. \end{aligned} \tag{4.30}$$

The results in Table 2(b) confirm that for our parameterization scaled consumption and human wealth both increase, i.e. the effects in (4.28) and (4.29) dominate the combined propensity effect (4.27) and growth effect (4.30).

In Figure 1 the dashed lines depict the life-cycle profiles associated with the model featuring imperfect annuities. Scaled consumption is hump-shaped but peaks at a rather high age.[3] The profiles for scaled financial assets, wages, and pension payments are all very similar to the ones for the core model.

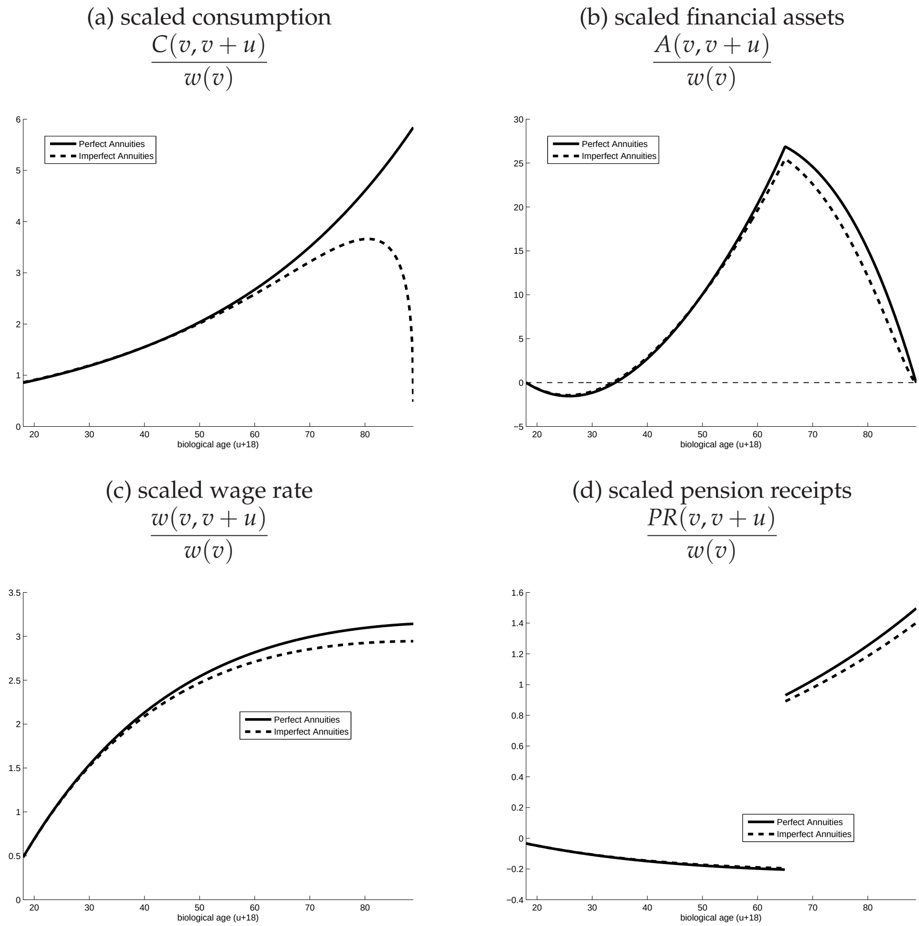
4.3 Ageing: the big picture

In this section we put our model to work on the big policy issue of demographic change. Population ageing remains one of the key issues in economic policy in the Netherlands. During the 2010 Dutch parliamentary election campaign numerous parties went so far as to call future policy on pensions and the retirement age a breaking point for the post-electoral coalition scramble. In this section we look at the big picture and study the effect of ageing and demographic change on the steady-state rate of economic growth of a country.[4]

[3] Bütler (2001) and Hansen and İmrohoroğlu (2008) also find that the hump occurs too late in life. Alessie and de Ree (2009, p. 113) decompose Dutch consumption into durables and non-durables. They find that non-durable consumption peaks at age 45 whereas durable consumption reaches its maximum at about age 41.

[4] For an accessible survey of the literature on the topic of population ageing and economic growth, see Bloom et al. (2008). Recent contributions using the endogenous growth framework include Fougère and

Figure 4.1. Life-cycle profiles and the role of annuity imperfections



We start our analysis with some stylized facts for the Netherlands.[5] In the period 2005-10 the crude birth rate is about $\beta = 1.13\%$ per annum whereas for 2035-40 it is projected to change to $\beta = 1.05\%$ per annum. The population growth rates are, respectively, $\pi = 0.41\%$ per annum for 2005-10 and $\pi = -0.01\%$ per annum 2035-40. Finally, the old-age dependency ratio is, respectively 23% in 2010 and 46% in 2040. We wish to simulate our model using a demographic shock which captures the salient features of these stylized facts. Since we restrict attention to steady-state comparisons in this paper, we make the strong assumption that the country finds itself in a demographic steady state both at present and in 2040.

4.3.1 A demographic shock

The demographic shock that we study is as follows.[6] First, we assume that the population growth rate changes from $\pi_0 = 0.5\%$ to $\pi_1 = 0\%$ per annum. Second, we use our estimated demographic process (4.8) but change the η_1 parameter in such a way that an old-age dependency ratio of 46% is obtained. Writing $e^{-M_i(u)} \equiv (\hat{\eta}_0 - e^{\eta_{1,i}u}) \,/\, (\hat{\eta}_0 - 1)$ the old-age dependency ratio can be written as:

$$dr\left(\pi_i, \eta_{1,i}\right) \equiv \frac{\int_{47}^{\bar{D}_i} e^{-\pi_i s - M_i(s)} ds}{\int_0^{47} e^{-\pi_i s - M_i(s)} ds}, \tag{4.31}$$

where $\bar{D}_i \equiv \left(1/\eta_{1,i}\right) \ln \eta_0$. Using this expression we find that η_1 changes from $\eta_{1,0} = \hat{\eta}_1 = 0.0680$ to $\eta_{1,1} = 0.0581$. The associated values for the crude birth rate are obtained by imposing the suitably modified demographic steady-state condition. We find that β changes in the model from $\beta_0 = 0.0204$ to $\beta_1 = 0.0151$. Figure 2(a) shows that the new instantaneous mortality profile shifts to the right. Figure 2(b) illustrates the change in the population composition. In the new steady state, the population distribution features less mass at lower ages and more at higher ages, i.e. the population pyramid becomes narrower and higher.

Mérette (1999), Futagami and Nakajima (2001), Heijdra and Romp (2006), and Prettner (2009).

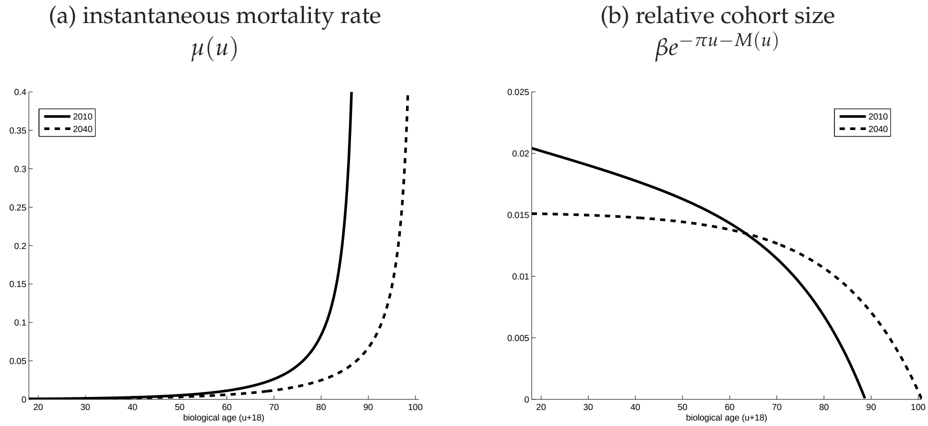
[5] These figures are taken from United Nations, *World Population Prospects: The 2008 Revision Population Data Base*, `http://esa.un.org/unpp`. We use data for the medium variant.

[6] In this chapter we focus on demographic changes induced by a change in the mortality rate. When we return to the analysis of demographic changes in Chapter 6 we study how demographic changes induced by a change in the birth rate differ from demographic changes induced by a change in the mortality rate.

The effect on the economic growth rate of the demographic shock depends critically on the type of pension system. We consider three scenarios. In the first scenario the pension system is DC, the contribution rate and retirement age are kept constant ($\theta_0 = 0.07$ and $R_0 = 47$), pension payments to the elderly are reduced to balance the budget of the PAYG system. Columns (c)–(d) in Table 2 report the results for the two cases with perfect (PA) and imperfect annuities (PA). Since the effects are qualitatively the same for PA and IA, we restrict attention to the latter case. Comparing columns (b) and (d) several features stand out. First, the ageing shock has a large effect on the supply of (efficiency units of) labour, i.e. n falls by more than fifteen percent. This is an obvious consequence of the fact that the population proportion of working-age persons declines. Second, the pension payments to retirees are almost halved. Third, notwithstanding the decrease in pensions, scaled consumption and human wealth at birth both increase dramatically. More people expect to survive into retirement and, once retired, the period of retirement is also increased. Fourth, the macroeconomic growth rate increases dramatically, from 1.91% to 3.27% per annum. The intuition behind this strong growth effect can be explained with the aid of Figure 3. The solid lines represents the core case of Table 2(b) and the dashed lines illustrate the results from Table 2(d). Following the demographic shock scaled consumption is uniformly higher and peaks at a later age. Scaled financial assets are somewhat lower during youth but much higher thereafter. As Figure 3(b) shows there is a huge savings response which explains the large increase in the macroeconomic growth rate. In conclusion, of the main growth channels identified by Bloom et al. (2008, p. 2), labour supply falls (and thus retards growth) but the capital accumulation effect is so strong as to lead to a strong positive effect of longevity on economic growth.

In the second scenario the pension system is DB, the pension payments and retirement age are kept constant ($\zeta_0 = 0.3632$ and $R_0 = 47$), and pension contributions by the young are increased to balance the budget of the PAYG system. Columns (e)–(f) in Table 2 give the results for this case. Comparing columns (b), (d) and (f) the following features stand out. First, the contribution rate increase is quite substantial, it almost doubles from $\theta_0 = 0.07$ to $\theta_1 = 0.1394$. Second, though scaled consumption, scaled human wealth, and the economic growth rate are higher than in the base case, they are lower than under the DC scenario. As Figure 3 shows, the capital accumulation effect

Figure 4.2. Demographic shock



of the longevity shock is substantially dampened under a DB system. Intuitively, by taking from the young and giving to the old the PAYG system redistributes from net savers to net dissavers.

Finally, in the third scenario both θ and ζ are kept at their pre-shock levels and the retirement age is increased to balance the budget of the PAYG system. Columns (g)–(h) in Table 2 give the results for this case. Comparing columns (b), (d), (f), and (h) the following features stand out. First, under the retirement age (RA) scenario the longevity shock necessitates an increase in the biological retirement age 65 to 75.3 years. i.e. the value of R restoring budget balance changes from $R_0 = 47$ to $R_1 = 57.3$. Second, compared to the DB and DC cases, labour supply increases strongly in the RA scenario. Third, the economic growth rate, though still higher than in the base case, is slightly lower that under DB and much lower than under DC. The intuition behind this result is clear from Figure 3(b) which shows that the savings response following the longevity shock is lower than either DB or DC.

The negative relationship between the retirement age and economic growth is surprising in light of the current (Dutch) policy debate in which an increase of the retirement age has become the paradigm for weathering the generational storm (see, for instance, Bovenberg and Gradus (2008)). This finding, however, fits well with the analysis of Bloom *et al.* (2007) who show that the recent increase in adult mortality has increased the savings rate in countries where the pension system contains a strong incentive to retire at the early eligibility age.[7]

The contrast between the findings from the literature and the policy debate is that the policy debate is predominantly occupied with the sustainability of government finances, which have come under pressure due to the additional influx of elderly into the receiving end of public pensions. The question arises whether the improvement of government finances due to an increase in the retirement age is not simply bought against a decreased incentive to save? We leave this interesting trade-off between government finances and the savings rate as an issue for future research because our current model is not equipped to give a compelling answer. However, we do emphasize that the analysis in this chapter highlights the fact that focusing solely on the sustainability of government finances may induce a policy with adverse long-run repercussions.

4.3.2 Robustness

The clear message emerging from the discussion so far is that the type of pension system in place has a quantitatively large influence on the link between longevity and macroeconomic growth. Indeed, the same longevity shock can either lead to a huge increase in growth (under DC) or only a modest increase (under RA). But how robust are these conclusions? As is pointed out by Bloom *et al.* (2008, p. 3), "population data are not sacrosanct" and UN predictions are revised substantially over time. In short, our stylized demographic facts may be more like "factoids".[8]

We study the robustness issue in Table 3. We restrict attention to the case with imperfect annuities, and column (a) in the table represents the base case. It coincides with the pre-shock steady state reported in Table 3(b). Columns (b)–(c) in Table 3 report the results under the DC scenario for alternative demographic shocks. In contrast, columns (d)-(e) show how a much more broadly defined PAYG system reacts to the original demographic shock under DC, DB, and RA.

[7] In accordance with our other findings Bloom *et al.* (2007) also find that positive impact of ageing on economic growth is mitigated if a PAYG system with high benefits is in place.

[8] De Waegenaere *et al.* (2010) provide a survey of the recent literature on longevity risk (i.e. the risk that mortality predictions turn out to be wrong). In accordance with Bloom *et al.* (2008) they show that estimates on future mortality rates differ substantially and depend on a plethora of uncertain factors.

In column (b) we assume that the old-age dependency ratio is 30% rather than 46% in 2040. As in the original shock we continue to assume that $\pi_1 = 0\%$ per annum. By using (4.31) we obtain new values for the demographic parameters, i.e. $\eta_{1,1} = 0.0662$ and $\beta_1 = \bar{\mu}_1 = 0.0172$. The alternative demographic shock causes a small increase in the economic growth rate. Whereas the original demographic shock caused growth to increase from 1.91% to 3.27% per annum (See Table 2, columns (b) and (d)), the alternative one only raises the growth rate to 2.33% per annum. The alternative ageing shock is relatively small, and pensions are reduced much less drastically than under the original demographic shock. The private savings response is quite small as a result.

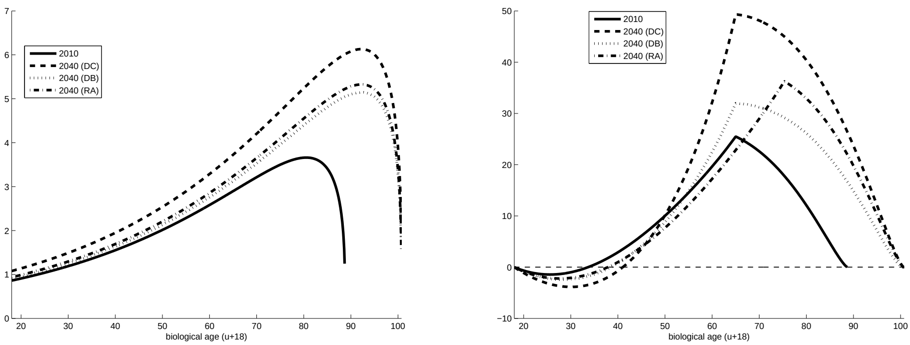
In column (c) we keep the dependency ratio at 46% but assume that the population growth rate is 0.5% rather than 0% per annum in 2040. Under this assumption the demographic parameters are equal to $\eta_{1,1} = 0.0540$, $\beta_1 = 0.0168$, and $\bar{\mu}_1 = 0.0118$. This type of demographic shock produces a huge increase in the macroeconomic growth rate. The intuition is the same as before – see the discussion relating to Table 2(d) above. The large growth effect is all the more surprising in view of the growth equation (T1.6) which directly features $-\pi$ on the right-hand side. So even though the demographic shock itself retards growth by 0.5% per annum, the huge private savings response more than compensates for this effect.

In conclusion, the two alternative demographic shocks give rise to qualitatively the same predictions as we obtained for the original shock. Under a DC system economic growth is boosted because the labour supply effect is strongly dominated by the capital accumulation effect.

As a final robustness check we investigate whether the *size* of the PAYG system influences the relationship between longevity and economic growth. We return to the original demographic shock featuring $\pi_1 = 0.05\%$ per annum and an old-age dependency ratio of 46% ($\eta_{1,1} = 0.0581$ and $\beta_1 = 0.0151$). As was pointed out by Broer (2001, p. 89), "in an ageing society, both the health insurance system and the pension system impose an increasing burden on households. ... Thus as the share of elderly in the population grows, the contribution base [of the public health insurance system, JM] shrinks at the same time when demand for health care increases." In short, it can be argued that the public health insurance system itself contains elements of a PAYG type, i.e. it taxes the young (and healthy) and provides resources to the old (and infirm).

Figure 4.3. Life-cycle profiles before and after the demographic shock

(a) scaled consumption $\dfrac{C(v, v+u)}{w(v)}$ (b) scaled financial assets $\dfrac{A(v, v+u)}{w(v)}$



Whereas it is beyond the scope of the present paper to fully model the health insurance system, we take from Broer's analysis the idea that the PAYG system may be broader than just the public pension system itself. We study the quantitative consequences of PAYG system size in columns (d)–(g) in Table 3. Column (d) shows what happens to the initial steady-state economy if the contribution rate is increased from $\theta_0 = 0.07$ to $\theta_1 = 0.15$. The comparison between columns (a) and (d) reveals that there is a huge drop in the growth rate, from $g = 1.91\%$ to $g = 1.19\%$ per annum. Intuitively the larger PAYG system takes more from the young and gives more to the old. This chokes off private savings and retards economic growth.

Columns (e)–(g) in Table 3 shows the effects of the original demographic shock under DC, DB, and RA. The growth increases under all scenarios with the largest effect occurring under the DC system. Interestingly, whereas the growth effect was smallest for the RA case in the original model with the narrowly defined PAYG system, for a large PAYG system it is smallest for the DB scenario.

4.3.3 Limitations

A few words of caution are in place when interpreting our conclusions. There are several limitations. First, our analysis consists of steady-state comparisons and space con-

Table 4.3. Alternative scenarios

	Initial PAYG system			**Large PAYG system**			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
		DC	DC		DC	DB	RA
$\frac{C(v,v)}{w(v)}$	0.8609	0.9268	1.0971	0.7047	0.8818	0.6109	0.7678
$\frac{H(v,v)}{w(v)}$	27.0207	29.4510	38.0243	22.1187	29.6744	20.5594	25.8384
g (%)	1.91	2.33	3.43	1.19	2.59	1.51	1.71
n	0.9675	0.9217	0.8155	0.9675	0.8212	0.8212	0.9589
$\frac{w(t)}{k(t)}$	0.2894	0.3038	0.3434	0.2894	0.3410	0.3410	0.2920
$\frac{c(t)}{w(t)}$	1.0570	1.0094	0.8465	1.0817	0.8918	0.9236	1.0715
ζ	0.3632	0.2796	0.1812	0.7783	0.3910	0.7783	0.7783
z	0.0200	0.0200	0.0111	0.0174	0.0130	0.0120	0.0148
θ						0.2986	
$R+18$							75.3

Notes. Column (a) is the core model with imperfect annuities (column (b) in Table 2). Column (b) dependency ratio in 2040 equal to 30% instead of 46%. Column (c) population growth rate in 2040 equal to 0.5% instead of 0% per annum. Column (d) bigger PAYG system ($\theta = 0.15$). Columns (e)–(f) show the effects of the original demographic shock for the large PAYG system under DC and DB. Column (g) leaves θ and ζ unchanged and features a higher retirement age.

siderations prevent us from studying the transitional dynamics of a longevity shock. Although we find that in the steady state a longevity shock has beneficial effects on growth, it need not be the case that transition is monotonic. Second, we have merely analyzed growth but not individual welfare. However, as we assume exogenous labour supply, higher growth automatically translates into higher welfare because discounted income of individuals increases. Third, we have assumed that labour supply and the retirement age are exogenous. We have chosen this approach here in order to keep the model as simple as possible. Indeed, endogenization of both the hours decision over the life cycle and/or the retirement date is fairly straightforward – see e.g. Heijdra and Romp (2009) and Chapter 2 and 3 above. Fourth, we have ignored aggregate risk and the risk-sharing properties of pension systems. The interested reader is referred to Bovenberg and Uhlig (2008) who apply a two-period stochastic overlapping generations model featuring endogenous growth to study the consequences of particular pension systems on risk-sharing between generations. Finally, we have studied a closed economy. This is not a convincing representation of the Dutch economy which is extremely open and small in world markets. However, aging is a global phenomenon. Hence, to model a small open economy with fixed factor prices is equally unconvincing. Here we have chosen the closed economy framework to zoom in on the global consequences of ageing on capital accumulation and economic growth – the big picture.

4.4 Conclusions

In this chapter we apply the model developed in the previous chapters to study the relationship between aging and economic growth and the mediating role that government policy has on this relationship. We find that, in principle, aging increases the economic growth rate. However, if a defined benefit system is in place the growth effect weakens somewhat because of the increase in the contribution rate necessary to finance the additional pensioners. In order to circumvent this adverse effect on the growth rate, the government might consider to switch to a defined contribution system or to increase the retirement age. Surprisingly, we find that the latter policy option has adverse effects on the economy.

This chapter completes the analysis of various economic issues based on the model developed in Chapter 2. In the initial chapter we set out to analyse what the impact is of imperfect annuity markets on individual decisions and macroeconomic outcomes. In further analysis we found that the model is very versatile and can be used to study two important issues at the forefront of economic policy making. In Chapter 3 we applied to model to issues of taxation and in Chapter 4 to an analysis of pension systems. In the next chapter we return to the issue of annuities by asking whether opening up an annuity market is also welfare enhancing from the macroeconomic perspective. In Chapter 6 we close the analysis by returning to the issue of demographic change and study how different sources of demographic change affect the aggregate capital stock.

Chapter 5

The tragedy of annuitization*

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* This chapter is based on Heijdra, Mierau and Reijnders (2010).

5.1 Introduction

A recurring theme of the previous chapters has been that, in the presence of longevity risk, life annuties are very attractive insurance instruments. Intuitively, annuities allow for risk sharing between lucky (long-lived) and unlucky (short-lived) individuals (Kotlikoff et al., 1986). These increased risk-sharing opportunities ensure that life annuities are welfare maximizing from a microeconomic perspective.

From a macroeconomic perspective, however, it is not immediately clear whether or not annuities are welfare improving. There are two key mechanisms that are ignored in a microeconomic analysis. First, in the absence of private annuities there will be accidental bequests which, provided they are redistributed in one way or another to surviving agents, boost the consumption opportunities of such agents. See, among others, Sheshinski and Weiss (1981), Abel (1985), Pecchenino and Pollard (1997), and Fehr and Habermann (2008) on this point. Second, the availability of annuities affects the rate of return on an individual's savings. As a result, aggregate capital accumulation will generally depend on whether or not annuity opportunities are available. Capital accumulation in turn determines wages and the interest rate if factor prices are endogenous.

The objective of this chapter is to study the general equilibrium effects of life annuities. Our model has the following features. First, we postulate a simple general equilibrium model of a closed economy. On the production side we allow for a capital accumulation externality of the form proposed by Romer (1989). The production side of the model is quite flexible in that it can accommodate both the exogenous and the endogenous growth models as special cases.

Second, and in contrast to the previous chapters, we assume that the economy is populated by overlapping generations of two-period-lived agents facing longevity risk. Just as in the Diamond (1965) model, life consists of two phases, namely youth and old age, but unlike that model there is a positive probability of death at the end of youth. At birth, agents are identical in the sense that they feature the same preferences, have the same labour productivity, and face the same death probability.

From the perspective of the previous chapters the switch to a two-period model allows us to give an analytical description of the transition as well as the steady-state effects. Naturally, this benefit comes at the cost of not being able to give the same degree of detail at the individual level. An interesting alley for future research is to combine the models from chapters 2-4 with the model in the current chapter to come to a complete description of the consequences of opening up an annuity market in an elaborately specified general equilibrium model.

Third, in the absence of annuities we assume that the resulting accidental bequests flow to the government. We investigate the general equilibrium effects of three prototypical revenue recycling schemes. In particular, the policy maker can (a) engage in wasteful expenditure (the WE scenario), (b) give lump-sum transfers to the old (the TO scenario), or (c) provide lump-sum transfers to the young (the TY scenario).

Fourth, we compare the different revenue recycling schemes with the case in which annuities are available. In particular, we assume that private annuity markets are perfectly competitive. With perfect annuities (the PA scenario) the probability of death determines the wedge between the rate of return on physical capital and the annuity rate of return. Since the latter exceeds the former, rational non-altruistic individuals fully annuitize their savings.

The main finding of the chapter concerns the phenomenon which we call the *tragedy of annuitization:* although full annuitization of assets is privately optimal it may not be socially beneficial due to adverse general equilibrium repercussions. If all agents invest their financial wealth in the annuity market, then the resulting long-run equilibrium leaves everyone worse off compared to the case where annuities are absent and accidental bequests are redistributed to the young (or even wasted by the government). In the exogenous growth model we demonstrate the existence of two versions of the tragedy. In the *strong* version, opening up perfect annuity markets in an economy in which accidental bequests initially go to waste (switch from WE to PA) results in a decrease in steady-state welfare of newborns. Interestingly, this rather surprising result holds for a reasonable (i.e. low) value of the intertemporal substitution elasticity. In such a case the beneficial effects of annuitization are more than offset by a substantial drop in the long-run capital intensity and in wages. Future newborns would have been better off if no annuity markets had been opened.

There is also a *weak* version of the tragedy in the exogenous growth model. If the economy is initially in the equilibrium with accidental bequests flowing to the young, then opening up annuity markets will reduce steady-state welfare regardless of the magnitude of the intertemporal substitution elasticity. Intuitively, private annuities redistribute assets from deceased to surviving elderly in an actuarially fair way whereas transferring unintended bequests to the young constitutes an intergenerational transfer. This intergenerational transfer induces beneficial savings effects, which, in the end, lead to higher welfare.

In the endogenous growth model and restricting attention to realistic values for the intertemporal substitution elasticity, both versions of the tragedy show up in terms of the macroeconomic growth rate. Growth is highest in the TY case, and the rate under the WE case exceeds the one for the PA scenario.

In light of the finding that the introduction of annuities decreases the macroeconomic growth rate it is interesting to briefly reflect on the findings in Chapter 2 and 3. In Chapter 2 we saw that the growth rate of the economy decreases if annuities are not priced in an actuarially fair way. In Chapter 3 we revisited this result and showed that if the profits made by the annuity firms are redistributed with a skew toward the young the negative impact on growth is partly mitigated. The difference between the results in the current and the previous chapters can be traced back to the exact redistribution structure used in the two models. In the model of Chapter 3 the redistribution has a *skew* toward younger generations whereas the model in this chapter redistributes *all* the funds to the newborns. This difference suggests that there is a combination of the TO and TY redistribution scheme in which the negative growth effects can be exactly off-set or even disappear. In future research it would be interesting to study where this turning point lies and how it is determined.

The structure of the chapter is as follows. Section 2 presents the model in its most general form. Section 3 studies the analytical properties of the exogenous growth version of the model. It also computes, both analytically and quantitatively, the allocation and welfare effects of scenario switches. Section 4 is the core of the chapter. It shows what happens to allocation and welfare if a perfectly competitive annuity market is opened up at some point in time. It also highlights the importance of initial conditions, i.e. it demonstrates that the results depend not only on the availability of annuities but also on the scenario that is replaced by these insurance markets. Section 5 briefly discusses the effects of annuitization in the endogenous growth version of the model. Section 6 restates the main results and presents some possible extensions. All mathematical results are collected in a seperate appendix and can be found in Heijdra, Mierau and Reijnders (2010).

5.2 The model

5.2.1 Consumers

Each agent lives for a maximum of two periods and faces a positive probability of death between the first and the second period. Agents work full-time during the first period of their lives (labeled "youth") and – if they survive – retire in the second period ("old age"). The expected lifetime utility of an individual born at time t is given by:

$$\mathbb{E}\Lambda_t^y \equiv U(C_t^y) + \frac{1-\pi}{1+\rho} U(C_{t+1}^o), \tag{5.1}$$

where C_t^y and C_{t+1}^o are consumption during youth and old age, respectively, $\rho > 0$ is the pure rate of time preference, and $\pi > 0$ is the probability of death. Individuals have no bequest motive and, therefore, attach no utility to savings that remain after they die. We assume that the utility function is of the constant relative risk aversion (CRRA) type:

$$U(C) = \begin{cases} \dfrac{C^{1-1/\sigma} - 1}{1 - 1/\sigma} & \text{if } \sigma > 0,\ \sigma \neq 1, \\ \ln C & \text{if } \sigma = 1, \end{cases} \tag{5.2}$$

where σ is the elasticity of intertemporal substitution. The agent's budget identities for youth and old age are given by:

$$C_t^y + S_t = w_t + Z_t^y, \tag{5.3a}$$

$$C_{t+1}^o = Z_{t+1}^o + (1 + r_{t+1}) S_t, \tag{5.3b}$$

where w_t is the wage rate, r_t is the interest rate, S_t denotes the level of savings, and Z_t^y and Z_{t+1}^o are transfers received from the government during either youth or old age (see below). Combining the equations in (5.3) yields the consolidated lifetime budget constraint:

$$C_t^y + \frac{C_{t+1}^o}{1+r_{t+1}} = w_t + Z_t^y + \frac{Z_{t+1}^o}{1+r_{t+1}}. \tag{5.4}$$

If an agent dies before reaching old age his savings flow to the government in the form of an accidental bequest. Due to mortality risk agents are not allowed to hold negative savings (i.e. loans). In case of premature death their loans would be unaccounted for.

The agent chooses C_t^y, C_{t+1}^o and S_t in order to maximize expected lifetime utility (5.1) subject to the budget constraint (5.4) and a non-negativity constraint on savings. Assuming an interior optimum ($S_t > 0$), the agent's optimal plans are fully characterized by:

$$C_t^y = \Phi\left(r_{t+1}\right)\left[w_t + Z_t^y + \frac{Z_{t+1}^o}{1+r_{t+1}}\right], \tag{5.5}$$

$$\frac{C_{t+1}^o}{1+r_{t+1}} = \left[1-\Phi\left(r_{t+1}\right)\right]\left[w_t + Z_t^y + \frac{Z_{t+1}^o}{1+r_{t+1}}\right], \tag{5.6}$$

$$S_t = \left[1-\Phi\left(r_{t+1}\right)\right]\left[w_t + Z_t^y\right] - \Phi\left(r_{t+1}\right)\frac{Z_{t+1}^o}{1+r_{t+1}}, \tag{5.7}$$

where $\Phi(r_{t+1})$ is the marginal propensity to consume out of total wealth (wage income and transfers) in the first period:

$$\Phi\left(r_{t+1}\right) \equiv \left[1 + \left(\frac{1-\pi}{1+\rho}\right)^{\sigma}\left(1+r_{t+1}\right)^{\sigma-1}\right]^{-1}, \qquad 0 < \Phi\left(\cdot\right) < 1. \tag{5.8}$$

Note that the impact of a change in the future interest rate on current savings is fully determined by the elasticity of intertemporal substitution σ. For the special case with $\sigma = 1$ (logarithmic utility) savings are completely independent of the interest rate.[1] The income effect of a higher interest rate is exactly offset by the substitution effect induced by a lower price of second period consumption. In the more general case with $\sigma > 1$ savings increase as the interest increases because the substitution effect dominates the income effect. If, on the other hand, $\sigma < 1$ the income effect is stronger than the substitution effect and savings decline as the interest rate rises.[2]

[1] If the government provides transfers to the old ($Z_{t+1}^o > 0$) there is also a positive human wealth effect on saving. In this paper, however, such transfers are proportional to the interest factor, $1 + r_{t+1}$, so that this human wealth effect is not operative. If the agent would also work in old age then the human wealth effects would result in an increase in the savings elasticity.

[2] From the empirical perspective the most relevant case appears to be the one with $0 < \sigma < 1$. See, for ex-

5.2.2 Demography

The population grows at an exogenous rate $n > 0$ so that every period a cohort of $L_t = (1+n) L_{t-1}$ young agents is born. In principle each generation lives for two periods, but not all of its members survive the transition from youth to old age. The total population at time t is equal to $P_t \equiv (1-\pi) L_{t-1} + L_t$.

5.2.3 Production

There is a constant and large number of identical and perfectly competitive firms. The technology available to each individual firm i is given by:

$$Y_{it} = \Omega_t K_{it}^{\alpha} N_{it}^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{5.9}$$

where Y_{it} is output, K_{it} is the employed capital stock, N_{it} is the amount of labour used in the production process, α is the capital share of output and Ω_t is the aggregate level of technology in the economy which is considered as given by individual firms. Factor demands of the individual firm are given by the following marginal productivity conditions:

$$w_t = (1-\alpha) \Omega_t k_{it}^{\alpha}, \tag{5.10a}$$

$$r_t + \delta = \alpha \Omega_t k_{it}^{\alpha-1}, \tag{5.10b}$$

where $k_{it} \equiv K_{it}/N_{it}$ is the capital intensity of firm i and $\delta > 0$ is the depreciation rate. Under the assumption of perfect competition in both factor markets all firms face the same factor prices and, therefore, they all choose the same level of capital intensity $k_{it} = k_t$.

Generalizing the insights of Pecchenino and Pollard (1997, p. 28) to a growing population we postulate that the inter-firm investment externality takes the following form:

$$\Omega_t = \Omega_0 k_t^{\eta}, \qquad 0 < \eta \leq 1-\alpha, \tag{5.11}$$

ample, Skinner (1985) and Attanasio and Weber (1995) who report estimates ranging between, respectively, 0.3 to 0.5, and 0.6 to 0.7.

where Ω_0 is a constant, $k_t \equiv K_t/N_t$ is the economy-wide capital intensity, $K_t \equiv \sum_i K_{it}$ is the total stock of capital and $N_t \equiv \sum_i N_{it}$ is the total labour force.

According to (5.11) total factor productivity increases in line with the aggregate capital intensity in the economy. That is, if an individual firm increases its capital stock, all firms benefit through a boost in the general productivity level Ω_t. The strength of this inter-firm investment externality is governed by the parameter η. If $0 \leq \eta < 1-\alpha$ then the long-run growth rate in per capita variables is exogenously determined and equal to zero. In the knife-edge case with $\eta = 1-\alpha$ the investment externality exactly offsets the decrease in marginal productivity following an addition to the capital stock. The aggregate production sector then exhibits single-sector endogenous growth of the type described in Romer (1989).

Using the general productivity index (5.11) we can write output (5.9) and factor prices (5.10) in aggregate terms:

$$y_t = \Omega_0 k_t^{\alpha+\eta}, \tag{5.12}$$

$$w_t = (1-\alpha)\,\Omega_0 k_t^{\alpha+\eta}, \tag{5.13}$$

$$r_t = \alpha\Omega_0 k_t^{\alpha+\eta-1} - \delta, \tag{5.14}$$

where $y_t \equiv Y_t/N_t$ is the level of output per worker and $Y_t \equiv \sum_i Y_{it}$ is aggregate output. We assume that the economy is sufficiently productive to assure a positive interest rate even when the investment externality attains its knife-edge value, i.e. $\alpha\Omega_0 > \delta$.

5.2.4 Government

The government administers the allocation of the accidental bequests, maintains a period-by-period balanced budget, and does not issue debt or retain funds. The government's budget constraint is therefore given by:

$$\pi\,(1+r_t)\,L_{t-1}S_{t-1} = (1-\pi)\,L_{t-1}Z_t^o + L_t Z_t^y + G_t. \tag{5.15}$$

That is, the total assets left behind by the agents who perish before reaching old age (left-hand side) are used to finance total transfers to the survivors Z_t^o, transfers to the newly arrived young Z_t^y, and unproductive government expenditure G_t.

We assume that the government can choose between two financing scenarios. Either it redistributes all its proceeds among the surviving agents in the form of lump-sum transfers or it uses the funds solely for unproductive government spending.

Transfer scenario. The government can either give the revenues exclusively to the young or exclusively to the old.[3]

(TY) If all the proceeds go to the young then $Z_t^o = G_t = 0$ in (5.15) and transfers to the young are given by:

$$Z_t^y = \frac{\pi \left(1 + r_t\right) L_{t-1} S_{t-1}}{L_t}. \tag{5.16a}$$

(TO) If all the transfers accrue to the elderly, both $Z_t^y = G_t = 0$ in (5.15) and transfers to the old are given by:

$$Z_t^o = \frac{\pi \left(1 + r_t\right) L_{t-1} S_{t-1}}{\left(1 - \pi\right) L_{t-1}}. \tag{5.16b}$$

Unproductive spending scenario.

(WE) If the full receipts from accidental bequests are used for unproductive government spending then $Z_t^y = Z_t^o = 0$ in (5.15) and wasteful government expenditures are:

$$G_t = \pi \left(1 + r_t\right) L_{t-1} S_{t-1}. \tag{5.16c}$$

5.2.5 Equilibrium

In equilibrium both factor markets must clear. As all young agents work full-time and all old agents are retired, the labour market equilibrium condition simply states that the total labour force must equal the total number of young agents, i.e. $N_t = L_t$. The capital market clearing condition implies that aggregate savings of the generation born at time $t-1$ must be equal to the total stock of productive capital in period t, i.e. $K_t = L_{t-1} S_{t-1}$. It immediately follows that, in equilibrium, the three revenue recycling

[3] Any convex combination of these two options is also feasible. We focus on the two extreme cases for ease of illustration.

scenarios implied by (5.16a)-(5.16c) above can be rewritten as:

$$Z_t^y = \pi (1 + r_t) k_t, \tag{5.17a}$$

$$Z_t^o = \frac{1+n}{1-\pi} \pi (1 + r_t) k_t, \tag{5.17b}$$

$$g_t = \pi (1 + r_t) k_t, \tag{5.17c}$$

where $g_t \equiv G_t / L_t$ are per worker government expenditures.

Substituting individual savings (5.7) into the capital market clearing condition and using the aggregate factor prices (5.13) and (5.14) provides the fundamental difference equation of the model:

$$(1+n) k_{t+1} = [1 - \Phi (r_{t+1})] \left[w_t + Z_t^y \right] - \Phi (r_{t+1}) \frac{Z_{t+1}^o}{1 + r_{t+1}}. \tag{5.18}$$

For future reference we summarize the system of equations that characterizes the macro-economic equilibrium in Table 1. Equations (T1.1)–(T1.3) are the consumption and saving demand functions, (T1.4) states the definition for the marginal propensity to consume, equations (T1.5) and (T1.6) are the factor prices, (T1.7) is the government budget constraint with capital market equilibrium imposed, and (T1.8) is the fundamental difference equation.

5.3 The exogenous growth model

In this section and the next we study the exogenous growth version of our model, i.e. we assume that the capital accumulation externality parameter satisfies $0 \leq \eta < 1 - \alpha$ so that there are diminishing returns to the macroeconomic capital stock. (The knife-edge model with $\eta = 1 - \alpha$ is briefly discussed in Section 5.5 below.) Throughout the chapter we assume that the steady-state interest rate exceeds the rate of population growth. Empirical support for this assumption is provided by Abel et al. (1987).

Assumption 5.1. *[Dynamic efficiency] For each scenario the corresponding steady-state interest rate $\hat{r}$ satisfies $\hat{r} > n$.*

Table 5.1. The general model

(a) Individual choices:

$$C_t^y = \Phi(r_{t+1}) \left[w_t + Z_t^y + \frac{Z_{t+1}^o}{1 + r_{t+1}} \right] \tag{T1.1}$$

$$\frac{C_{t+1}^o}{1 + r_{t+1}} = [1 - \Phi(r_{t+1})] \left[w_t + Z_t^y + \frac{Z_{t+1}^o}{1 + r_{t+1}} \right] \tag{T1.2}$$

$$S_t = [1 - \Phi(r_{t+1})] \left[w_t + Z_t^y \right] - \Phi(r_{t+1}) \frac{Z_{t+1}^o}{1 + r_{t+1}} \tag{T1.3}$$

$$\Phi(r_{t+1}) \equiv \left[1 + \left(\frac{1 - \pi}{1 + \rho} \right)^{\sigma} (1 + r_{t+1})^{\sigma - 1} \right]^{-1} \tag{T1.4}$$

(b) Factor prices and redistribution scheme:

$$r_t = \alpha \Omega_0 k_t^{\alpha + \eta - 1} - \delta \tag{T1.5}$$

$$w_t = (1 - \alpha) \Omega_0 k_t^{\alpha + \eta} \tag{T1.6}$$

$$\pi(1 + r_t) k_t = \frac{1 - \pi}{1 + n} Z_t^o + Z_t^y + g_t \tag{T1.7}$$

(c) Fundamental difference equation:

$$(1 + n) k_{t+1} = [1 - \Phi(r_{t+1})] \left[w_t + Z_t^y \right] - \Phi(r_{t+1}) \frac{Z_{t+1}^o}{1 + r_{t+1}} \tag{T1.8}$$

Definitions: Endogenous are C_t^y, C_{t+1}^o, S_t, r_{t+1}, w_t, k_t, and – depending on the redistribution scheme – one of Z_t^y or Z_t^o or g_t. Parameters: mortality rate π, population growth rate n, rate of time preference ρ, capital coefficient in the technology α, investment externality coefficient η, scale factor in the technology Ω_0, and depreciation rate of capital δ.

5.3.1 Stability and transition

We first study the dynamic properties of the model under the assumption that the government wastes the revenues from accidental bequests (the WE scenario). One of the crucial structural parameters is the intertemporal substitution elasticity, σ. Whilst the model can accommodate a wide range of values for σ, we nevertheless make the following assumption.

Assumption 5.2. *[Admissible values for σ] The intertemporal substitution elasticity satisfies:*

$$0 < \sigma \leq \bar{\sigma} \equiv \frac{2-\alpha-\eta}{1-\alpha-\eta}.$$

We defend this assumption on two grounds. First, the restriction is very mild. Indeed, empirical evidence suggests that σ falls well short of unity whereas – even in the absence of external effects ($\eta = 0$) – $\bar{\sigma}$ is much larger than unity. For example, for a capital share of $\alpha = 0.3$ we find that $\bar{\sigma} = 2.43$. In the presence of external effects ($\eta > 0$) $\bar{\sigma}$ is even larger. Second, by restricting the range of admissible values for σ the existence and stability proofs are simplified substantially.

The fundamental difference equation under the WE scenario can be written as follows:

$$[\Psi(k_{t+1}) \equiv] \ \frac{k_{t+1}}{1-\Phi(k_{t+1})} = \frac{(1-\alpha)\,\Omega_0}{1+n} k_t^{\alpha+\eta} \ \ [\equiv \Gamma(k_t)], \tag{5.19}$$

where $\Phi(k)$ is given by:[4]

$$\Phi(k) \equiv \left[1 + \left(\frac{1-\pi}{1+\rho}\right)^{\sigma} \left(1-\delta+\alpha\Omega_0 k^{\alpha+\eta-1}\right)^{\sigma-1}\right]^{-1}. \tag{5.20}$$

It is easy to show that $\Psi' > 0$ and $\Gamma' > 0$. We can prove the following proposition.

Proposition 5.1. *[Existence and stability of the WE model] Consider the WE model as given in (5.19)–(5.20) and adopt Assumption 5.2. The following properties can be established:*

(i) The model has two steady-state solutions; the trivial one features $k_{t+1} = k_t = 0$, and the

[4] Equation (5.20) is obtained by substituting (T1.5) into (5.8).

economically relevant satisfies $k_{t+1} = k_t = \hat{k}^{WE}$, *where* $\hat{k}^{WE}$ *is the solution to:*

$$\frac{\hat{k}^{WE}}{1-\Phi(\hat{k}^{WE})} = \frac{(1-\alpha)\,\Omega_0}{1+n}(\hat{k}^{WE})^{\alpha+\eta}.$$

(ii) The trivial steady-state solution is unstable whilst the non-trivial solution is stable:

$$0 < \frac{dk_{t+1}}{dk_t} < 1, \qquad \textit{for } k_{t+1} = k_t = \hat{k}^{WE}.$$

For any positive initial value the capital intensity converges monotonically to $\hat{k}^{WE}$.

Proof: See Heijdra *et al.* (2010b, Appendix A). □

We visualize the corresponding phase diagram in Figure 1(a) for different values of the intertemporal substitution elasticity. This figure is based on the following plausible parameter values that are used throughout much of the chapter. In the *benchmark* case we assume that the elasticity of intertemporal substitution is $\sigma = 1$ (i.e. log-utility), and that the investment externality is absent ($\eta = 0$). Each phase of life covers 40 years, the population grows by one percent per annum (so that $n = (1+0.01)^{40} - 1 = 0.49$), individuals face a probability of death between youth and old age of thirty percent ($\pi = 0.3$), the capital share of output is thirty percent ($\alpha = 0.3$), and the depreciation rate of capital is six percent per annum ($\delta = 0.92$). We set the production function constant and time preference rate such that output per worker is equal to unity and the interest rate is four percent per annum ($\hat{r} = 3.80$) in the WE scenario. We obtain $\Omega_0 = 2.29$ and $\rho = 3.47$ or 3.82% annually. The resulting steady-state values of the key variables of the model are given in Table 5.2(a).[5] Note that Assumptions 5.1 and 5.2 are both satisfied for this calibration.

In Figure 2(a) the solid line represents the fundamental difference equation (5.19) (for $\sigma = 1$) and the dotted line is the steady-state condition $k_{t+1} = k_t$.[6] The economically relevant steady-state equilibrium is at point E where the slope of (5.19) is strictly less than unity. Figure 1(b) plots $\Psi(k)$ (for different values of σ) and $\Gamma(k)$

[5] For different values of σ we re-calibrate the model (by choice of ρ and Ω_0) such that output and the interest rate remain the same in the WE scenario.

[6] The dash-dotted and dashed lines in Figure 1(a) represent the fundamental difference equation for different values of the intertemporal substitution elasticity, σ. Mathematically, these lines are described by $k_{t+1} = \Psi^{-1}(\Gamma(k_t))$.

separately. It conveniently illustrates the existence and stability properties of the two steady-state equilibria. In particular, it visualizes Proposition 5.1(ii) which proves that $\Gamma(k)$ is steeper (flatter) than $\Psi(k)$ around $k = 0$ ($k = \hat{k}$) for all feasible values of σ.

Suppose that at some time t the economy has converged to the steady-state implied by the WE scenario, i.e. $k_t = \hat{k}^{WE}$. What would happen at impact, during transition, and in the long run if the government were to switch to a transfer scenario? We study two such policy switches in turn, namely from WE to TO and from WE to TY.

Transfers to the old

The effects of a policy switch from the WE scenario to the TO scenario can be studied with the aid of the following fundamental difference equation:

$$[\Psi\left(k_{t+1}, z_1\right) \equiv] \; \frac{1 + z_1 \frac{\pi}{1-\pi} \Phi\left(k_{t+1}\right)}{1 - \Phi\left(k_{t+1}\right)} k_{t+1} = \Gamma\left(k_t\right), \tag{5.21}$$

where $\Gamma\left(k_t\right)$ is defined in (5.19) above, z_1 is a perturbation parameter ($0 \leq z_1 \leq 1$) and $\Psi\left(k_{t+1}, z_1\right)$ features positive partial derivatives $\Psi_k > 0$ and $\Psi_{z_1} > 0$. The case with $z_1 = 0$ is the WE scenario whilst for $z_1 = 1$ the TO case is obtained. The policy switch thus consists of a unit increase in z_1 occurring at time t in combination with the initial condition $k_t = \hat{k}^{WE}$. We provide the following proposition.

Proposition 5.2. *[Existence and stability of the TO model] Consider the TO model as given in (5.21) and adopt Assumption 5.2. The following properties can be established:*

(i) The model has two steady-state solutions; the trivial one features $k_{t+1} = k_t = 0$, and the economically relevant one satisfies $k_{t+1} = k_t = \hat{k}^{TO}$, where $\hat{k}^{TO}$ is the solution to:

$$\frac{1 + \frac{\pi}{1-\pi} \Phi(\hat{k}^{TO})}{1 - \Phi(\hat{k}^{TO})} \hat{k}^{TO} = \frac{(1-\alpha)\,\Omega_0}{1+n} (\hat{k}^{TO})^{\alpha+\eta}.$$

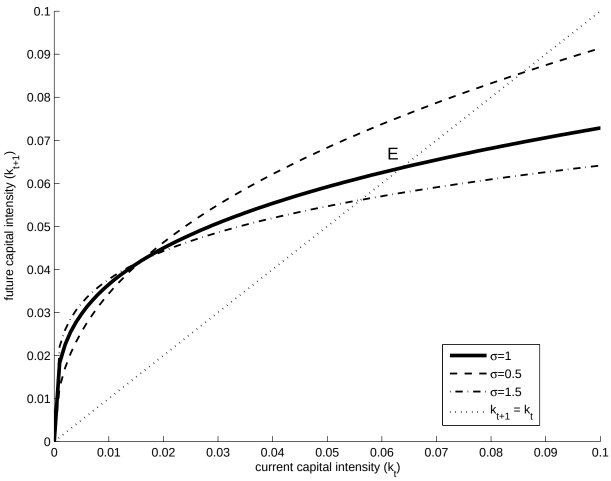
(ii) The trivial steady-state solution is unstable whilst the non-trivial solution is stable:

$$0 < \frac{dk_{t+1}}{dk_t} < 1, \qquad \text{for } k_{t+1} = k_t = \hat{k}^{TO}.$$

For any positive initial value the capital intensity converges monotonically to $\hat{k}^{TO}$.

Figure 5.1. Phase diagram and steady-state equilibria

(a) Phase diagram



(b) Steady-state equilibria

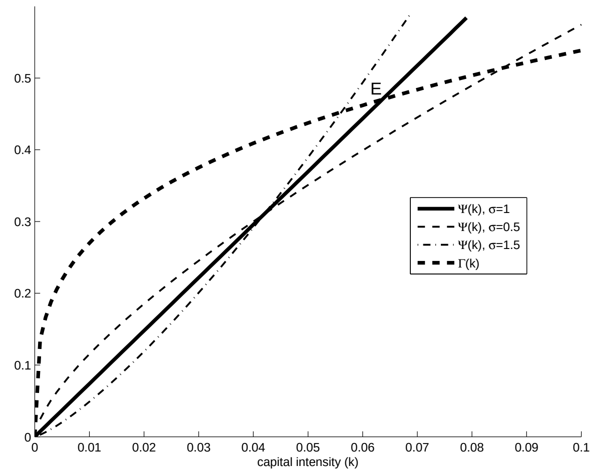


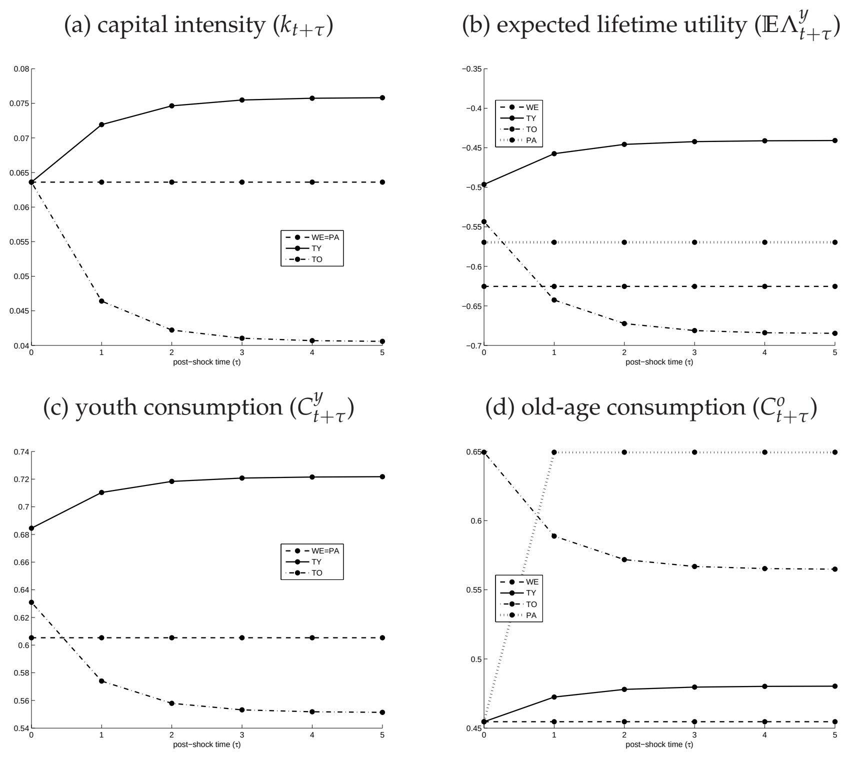
Table 5.2. Steady-state values with exogenous growth*

	Panel A: $\eta = 0, \sigma = 1$				Panel B: $\eta = 0, \sigma = \frac{1}{2}$				Panel C: $\eta = 0, \sigma = \frac{3}{2}$			
	(a) WE	(b) TO	(c) TY	(d) PA	(e) WE	(f) TO	(g) TY	(h) PA	(i) WE	(j) TO	(k) TY	(l) PA
$\hat{C}^y$	0.6053	0.5512	0.7218	0.6053	0.6053	0.5057	0.7393	0.5577	0.6053	0.5681	0.7145	0.6226
$\hat{C}^o$	0.4546	0.5647	0.4804	0.6495	0.4546	0.5040	0.5002	0.5741	0.4546	0.5893	0.4725	0.6815
$\hat{S}$	0.0947	0.0604	0.1129	0.0947	0.0947	0.0417	0.1284	0.0746	0.0947	0.0693	0.1071	0.1104
$\hat{Z}^o$		0.1694				0.1512				0.1768		
$\hat{Z}^y$			0.0968				0.1008				0.0952	
$\hat{y}$	1.0000	0.8736	1.0542	1.0000	1.0000	0.7821	1.0957	0.8877	1.0000	0.9105	1.0377	1.0472
$\hat{k}$	0.0636	0.0405	0.0758	0.0636	0.0636	0.0280	0.0862	0.0428	0.0636	0.0465	0.0720	0.0742
$\hat{w}$	0.7000	0.6115	0.7380	0.7000	0.7000	0.5474	0.7670	0.6214	0.7000	0.6374	0.7264	0.7330
$\hat{r}$	3.8010	5.5491	3.2541	3.8010	3.8010	7.4546	2.8954	5.3121	3.8010	4.9544	3.4106	3.3198
$\hat{r}_a$	4.00	4.81	3.69	4.00	4.00	5.48	3.46	4.71	4.00	4.56	3.78	3.73
$\hat{r}_a^A$				4.93				5.65				4.65
$\widehat{\mathbb{E}\Lambda}^y$	−0.6253	−0.6851	−0.4406	−0.5695	−0.7930	−1.0930	−0.4699	−0.8801	−0.5816	−0.5988	−0.4322	−0.5003

*Hats denote steady-state values. To facilitate interpretation, $\hat{r}_a$ and $\hat{r}_a^A$ are reported as annual percentage rates of return.

Figure 5.2. Transitional dynamics in the exogenous growth model

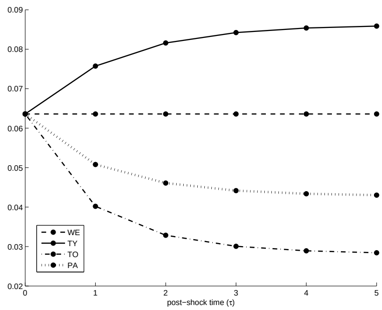
Panel A: Benchmark: $\sigma = 1$



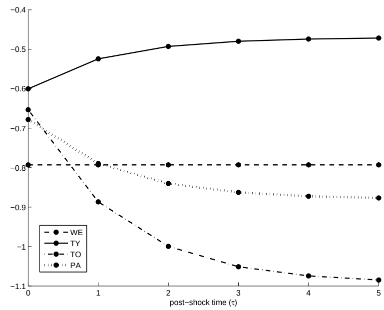
(Figure 5.2, continued)

Panel B: Weak intertemporal substitution effect: $\sigma = \frac{1}{2}$

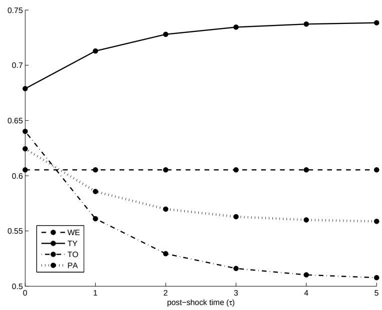
(e) capital intensity ($k_{t+\tau}$)



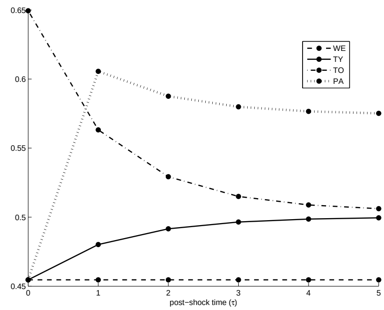
(f) expected lifetime utility ($\mathbb{E}\Lambda^{y}_{t+\tau}$)



(g) youth consumption ($C^{y}_{t+\tau}$)



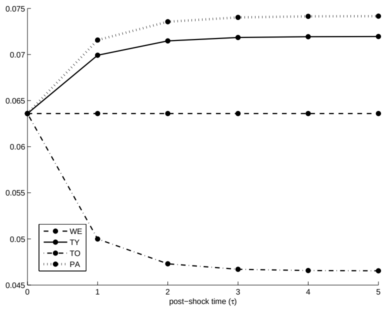
(h) old-age consumption ($C^{o}_{t+\tau}$)



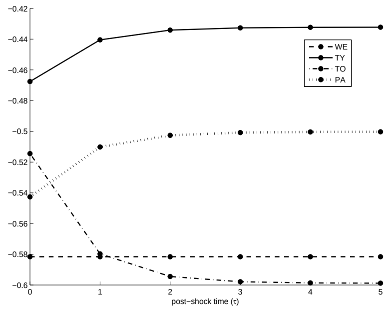
(Figure 5.2, continued)

Panel C: Strong intertemporal substitution effect: $\sigma = \frac{3}{2}$

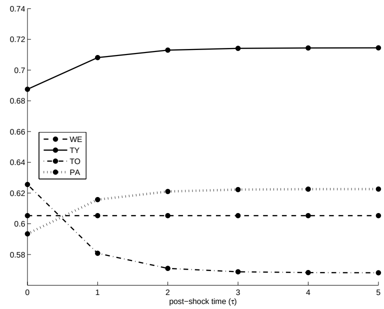
(i) capital intensity ($k_{t+\tau}$)



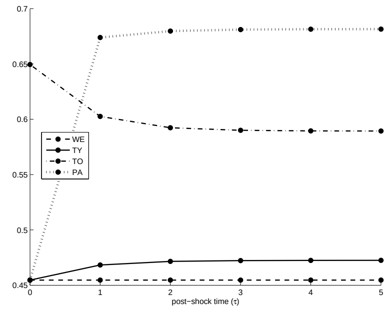
(j) expected lifetime utility ($\mathbb{E}\Lambda^{y}_{t+\tau}$)



(k) youth consumption ($C^{y}_{t+\tau}$)



(l) old-age consumption ($C^{o}_{t+\tau}$)



(iii) The steady-state capital intensity satisfies the following inequality:

$$0 < \hat{k}^{TO} < \hat{k}^{WE}.$$

Proof: See Heijdra *et al.* (2010b, Appendix B). □

For $\sigma = 1$ we visualize the transitional dynamics of the capital intensity in Figure 2(a) whilst the quantitative long-run results are reported in Table 2(b). In Figure 2(a) the horizontal axis records post-shock time τ and the vertical axis gives the values of $k_{t+\tau}$. By giving transfers to old agents, the old at the time of the policy switch ($\tau = 0$) are able to increase their consumption as they had not anticipated this windfall gain (see Figure 2(d)). The young at the time of the shock, however, react to the transfers they will receive in old age by reducing their saving below what it would have been under the WE scenario. This explains why the capital intensity drops substantially for $\tau = 1$ and beyond. Indeed, by using (5.21) we find the impact and long-run effects:

$$\left.\frac{dk_{t+1}}{dz_1}\right|_{k_t=\hat{k}^{WE}} = -\frac{\Psi_{z_1}}{\Psi_k} < 0, \qquad \left.\frac{dk_{t+\infty}}{dz_1}\right|_{k_t=\hat{k}^{WE}} = -\frac{\Psi_{z_1}}{\Psi_k - \Gamma'} < 0, \tag{5.22}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{TO}$. As the information in Table 2(a)–(b) reveals, compared to the WE scenario, long-run output per worker falls by almost thirteen percent under the TO case. Whereas the steady-state consumption profile is downward sloping under the WE scenario ($C^o < C^y$), it is upward sloping for the TO case ($C^o > C^y$). This result follows from the sharp increase in the interest rate that occurs in the TO scenario.[7]

Panels B and C in Table 2 and Figure 2 quantify and visualize the cases with, respectively, a weak intertemporal substitution effect (Panel B featuring $\sigma = \frac{1}{2}$) and a strong intertemporal substitution effect (Panel C featuring $\sigma = \frac{3}{2}$). The results are qualitatively the same as for the case with $\sigma = 1$. Quantitatively a relatively low (high) intertemporal substitution effect exacerbates (mitigates) the crowding-out effect on the capital intensity.

[7] The optimal consumption Euler equation is given by:

$$\frac{C^o_{t+1}}{C^y_t} = \left[\frac{(1-\pi)(1+r_{t+1})}{1+\rho}\right]^{\sigma}.$$

Transfers to the young

A policy switch from the WE case to the TY scenario can be studied with the following fundamental difference equation for the capital intensity:

$$\Psi\left(k_{t+1}\right)=\frac{\left[1-\alpha\left(1-z_{2}\pi\right)\right]\Omega_{0}k_{t}^{\alpha+\eta}+z_{2}\pi\left(1-\delta\right)k_{t}}{1+n}\quad\left[\equiv\Gamma\left(k_{t},z_{2}\right)\right],\tag{5.23}$$

where $\Psi\left(k_{t+1}\right)$ is defined on the left-hand side of (5.19), z_2 is a perturbation parameter $(0\leq z_2\leq 1)$ and $\Gamma\left(k_t,z_2\right)$ features positive partial derivatives $\Gamma_k>0$ and $\Gamma_{z_2}>0$. At time t there is a unit increase in z_2 and $k_t=\hat{k}^{WE}$ is the initial condition. We provide the following proposition.

Proposition 5.3. *[Existence and stability of the TY model] Consider the TY model as given in (5.23) and adopt Assumption 5.2. The following properties can be established:*

(i) The model has two steady-state solutions; the trivial one features $k_{t+1}=k_t=0$, and the economically relevant satisfies $k_{t+1}=k_t=\hat{k}^{TY}$, where $\hat{k}^{TY}$ is the solution to:

$$\frac{\hat{k}^{TY}}{1-\Phi(\hat{k}^{TY})}=\frac{\left[1-\alpha\left(1-\pi\right)\right]\Omega_{0}(\hat{k}^{TY})^{\alpha+\eta}+\pi\left(1-\delta\right)\hat{k}^{TY}}{1+n}.$$

(ii) The trivial steady-state solution is unstable whilst the non-trivial solution is stable:

$$0<\frac{dk_{t+1}}{dk_t}<1,\qquad \textit{for } k_{t+1}=k_t=\hat{k}^{TY}.$$

For any positive initial value the capital intensity converges monotonically to $\hat{k}^{TY}$.

(iii) The steady-state capital intensity satisfies the following inequality:

$$0<\hat{k}^{WE}<\hat{k}^{TY}.$$

Proof: See Heijdra *et al.* (2010b, Appendix C). □

For $\sigma=1$ we visualize the transitional dynamics of the capital intensity in Figure 2(a) whilst the quantitative long-run results are reported in Table 2(c). As Figure 2(a) shows, the capital intensity increases over time. By giving transfers to young agents only, the old at the time of the policy switch ($\tau = 0$) experience no effect at all. They just execute the plans conceived during their youth. In contrast, the shock-time young react to these transfers by increasing their saving above what it would have been under the WE scenario. This explains why the capital intensity increases dramatically for $\tau = 1$ and beyond – see the solid line in Figure 2(a). By using (5.23) we find the impact and long-run effects of the policy change on the capital intensity:

$$\left.\frac{dk_{t+1}}{dz_2}\right|_{k_t=\hat{k}^{WE}} = \frac{\Gamma_{z_2}}{\Psi'} > 0, \qquad \left.\frac{dk_{t+\infty}}{dz_2}\right|_{k_t=\hat{k}^{WE}} = \frac{\Gamma_{z_2}}{\Psi' - \Gamma_k} > 0, \tag{5.24}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{TY}$. As the information in Table 2(a) and (c) reveals, compared to the WE scenario, long-run output per worker increases by more than five percent under the TY case. Because the steady-state interest rate falls, the long-run consumption profile becomes more downward sloping than it was in the WE scenario.[8]

5.3.2 Welfare analysis

In this section we study the welfare implications of the different scenarios. With bounded externalities ($0 \leq \eta < 1 - \alpha$) consumption by young and old agents ultimately converges to time-invariant steady-state values. As a result we can compare the welfare effects of the separate regimes by focusing on the life-time utility of newborns, both along the transition path and in the steady state. The welfare effect for the old at the time of the shock follows trivially from their budget identity (5.3b), which can be rewritten as:

$$C_t^o = Z_t^o + (1 + r_t)(1 + n)\, k_t, \tag{5.25}$$

where we have used the fact that $S_{t-1} = (1 + n)\, k_t$. For the shock-time old agents all terms featuring in (5.25) are predetermined except the transfers to the old, Z_t^o, occurring exclusively in the TO scenario. Hence, C_t^o will not change following a policy change except if the switch is to the TO case.

The (indirect) lifetime utility function of current and future newborns can be writ-

[8] Panels B and C in Table 2 and Figure 2 confirm that the magnitude of σ affects the quantitative but not the qualitative conclusions.

ten as follows (for $\tau = 0, 1, \ldots$):

$$\mathbb{E}\Lambda^{y}_{t+\tau} \equiv \begin{cases} \dfrac{\Phi\left(r_{t+\tau+1}\right)^{-1/\sigma}\left(H^{y}_{t+\tau}\right)^{1-1/\sigma} - \dfrac{2+\rho-\pi}{1+\rho}}{1-1/\sigma} & \text{for } \sigma > 0,\ \sigma \neq 1 \\ \\ \Xi_0 + \dfrac{2+\rho-\pi}{1+\rho}\ln H^{y}_{t+\tau} + \dfrac{1-\pi}{1+\rho}\ln\left(1+r_{t+\tau+1}\right) & \text{for } \sigma = 1 \end{cases} \tag{5.26}$$

where Ξ_0 is a constant[9] and human wealth at birth of agents born τ periods after the policy change is given by:

$$H^{y}_{t+\tau} \equiv w_{t+\tau} + Z^{y}_{t+\tau} + \frac{Z^{o}_{t+\tau+1}}{1+r_{t+\tau+1}}. \tag{5.27}$$

The expressions in (5.25)–(5.27) are used to compute the transitions paths in Figures 2(b), (f), and (j) and the entries for $\widehat{\mathbb{E}\Lambda}^{y}$ in the final row of Table 5.2. For the analytical welfare effects at impact and in the long run, however, we employ the envelope theorem (see Heijdra *et al.*, 2010b). We consider each scenario in turn.

Transfers to the old

First we consider the welfare effects of a switch from the steady state of the WE case to the TO scenario. In what follows, $\hat{C}^{o}$, $\hat{C}^{y}$, $\hat{r}$, $\hat{w}$, and $\hat{k}$ denote steady-state values associated with the WE scenario. The welfare effect of the old at time t is equal to:

$$\frac{d\mathbb{E}\Lambda^{y}_{t-1}\left(z_1\right)}{dz_1} = \frac{1+n}{1+\rho}U'(\hat{C}^{o})\pi\left(1+\hat{r}\right)\hat{k} > 0. \tag{5.28}$$

The shock-time old are unambiguously better off because they receive a windfall transfer from the government. The welfare effect on the young at time t is more complicated because they can still alter their consumption and savings decisions in the light of the policy change. Although the wage rate faced by these agents is predetermined, their revised saving plans will induce a change in the future interest rate. After some ma-

[9] The definition of Ξ_0 is:

$$\Xi_0 \equiv \ln\left[\frac{1+\rho}{2+\rho-\pi}\right] + \frac{1-\pi}{1+\rho}\ln\left[\frac{1-\pi}{2+\rho-\pi}\right].$$

nipulation we find:

$$\frac{d\mathbb{E}\Lambda_t^y(z_1)}{dz_1} = U'(\hat{C}^y)(1+n)\hat{k}\left[\frac{\pi}{1-\pi} + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_1}\right] > 0. \tag{5.29}$$

The first term in square brackets represents the *direct effect* of the lump-sum transfer received at old age. Taken in isolation, this transfer expands the choice set and thus increases expected lifetime utility of shock-time newborns. The direct effect can be explained with the aid of Figure 3(a). The original budget line passes through E_0, which is the initial equilibrium. The shock-time young anticipate transfers in old age equal to Z_{t+1}^0. This shifts up the budget line in a parallel fashion.[10] Holding constant the initially expected future interest rate, the optimal point shifts from E_0 to E'. But this is not the end of the story because it is only the partial equilibrium effect.

The second term in square brackets on the right-hand side of (5.29) represents the *general equilibrium effect* of the policy switch. It follows from (5.22) that the future capital stock is lower and the interest rate is higher as a result of the switch. In terms of Figure 3(a), the budget line pivots in a clockwise fashion around point A_0 and optimal consumption moves from E' to E_1. At impact the general equilibrium effect thus brings about a further expansion of the choice set faced by the shock-time young. Not surprisingly, therefore, the change in welfare at impact is unambiguously positive for such agents. In terms of Figure 2(b), the dash-dotted line lies above the dashed line at post-shock time $\tau = 0$.

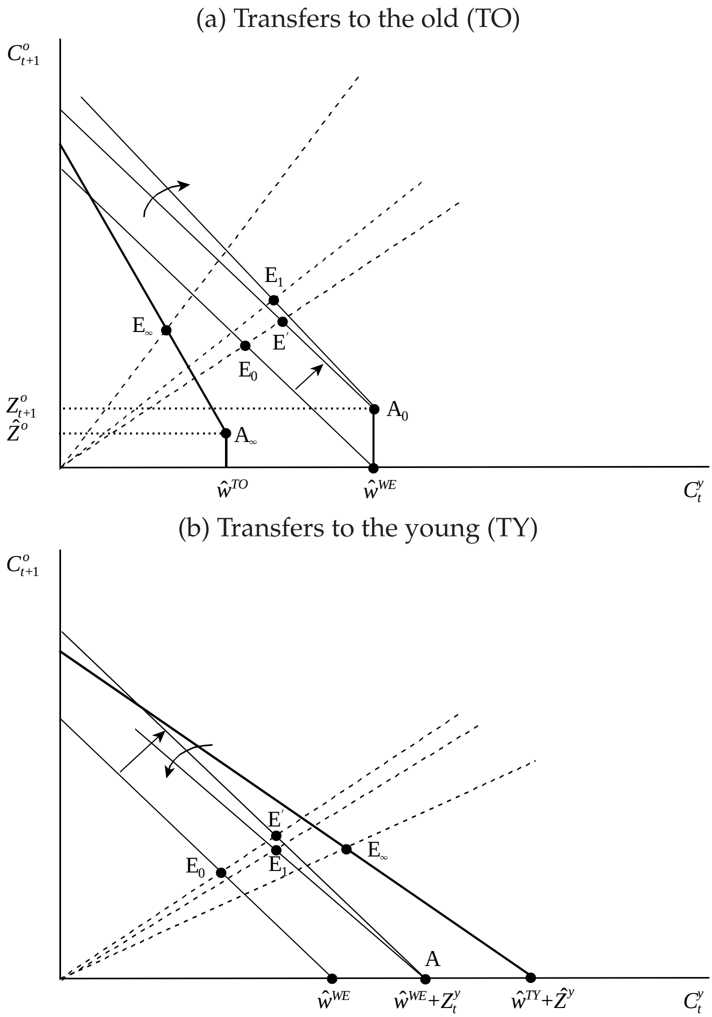
Before turning to the long-run welfare effects we first introduce the following lemma exploiting an important property of the factor-price frontier.

Lemma 5.1. *[Implications of the factor price frontier] Assume that* $0 \leq \eta < 1-\alpha$ *(exogenous growth model), the economy is initially in the steady state associated with the WE or TY scenario, and adopt Assumption 5.1 (*$\hat{r} > n$*, dynamic efficiency). Let* $dk_{t+\infty}/dz_i$ *denote the long-run effect on the capital intensity of a unit perturbation in* z_i *occurring at shock-time* $\tau = 0$ *and evaluated at* $z_i = 0$*. It follows that the long-run effect on weighted factor prices can be written as:*

$$\frac{\hat{C}^o}{(1+\hat{r})^2}\frac{dr_{t+\infty}}{dz_i} + \frac{dw_{t+\infty}}{dz_i} = \Delta\frac{dk_{t+\infty}}{dz_i}, \tag{L1.1}$$

[10] Remember that agents are not allowed to borrow and that, therefore, consumption bundles with $C_t^y > w_t$ remain unattainable.

Figure 5.3. Effect of government transfers in the exogenous growth model



where Δ is a positive constant:

$$\Delta \equiv \left[\eta + \alpha \left(1 - \alpha - \eta\right) \frac{\hat{r} - n}{1 + \hat{r}} \right] \frac{\hat{r} + \delta}{\alpha} > 0. \tag{L1.2}$$

Proof: See Heijdra *et al.* (2010b, Appendix E). □

The welfare effect experienced by future steady-state generations can be written as:

$$\frac{d\mathbb{E}\Lambda^{y}_{t+\infty}\left(z_1\right)}{dz_1} = U'(\hat{C}^y) \left[\frac{\pi\left(1+n\right)}{1-\pi} \hat{k} + \Delta \frac{dk_{t+\infty}}{dz_1} \right] \gtreqless 0, \tag{5.30}$$

where we have used Lemma 5.1 and note that $\lim_{\tau \to \infty} k_{t+\tau} = \hat{k}^{TO}$. The first term in brackets represents the steady-state direct effect, which is positive. The second term comprises the general equilibrium effect, which is negative because capital is crowded out in the long run (see (5.22) above). On the one hand the reduction in the long-run capital intensity increases the interest rate which positively affects welfare. But on the other hand it also reduces the wage rate, which lowers welfare. In terms of Figure 3(a), the budget line shifts to the left because of the fall in the long-run wage ($\hat{w}^{TO} < \hat{w}^{WE}$). In addition, long-run transfers are lower than anticipated transfers at impact ($\hat{Z}^o < Z^o_{t+1}$) so that point A_∞ lies south-west from A_0. The steady-state interest rate exceeds the future rate faced by shock-time newborns ($\hat{r}^{TO} > r_{t+1}$), i.e. the budget line is steeper than at impact. The steady-state equilibrium is at point E_∞.

Comparing columns (a) and (b) of Table 2 reveals that the long-run welfare effect of the policy switch is negative, i.e. the crowding out of capital induces a very strong reduction in wages which dominates the joint effect of the transfers and the interest rate. Ignoring agents who are alive at the time of the shock, it is thus better to let the accidental bequests go to waste than to give them to the elderly. To better understand the intuition behind this remarkable result, we first state the following lemma on the key features of the steady-state first-best social optimum (FBSO).

Lemma 5.2. *[Golden rules] Assume that $0 \leq \eta < 1 - \alpha$ (exogenous growth model), and define steady-state welfare of a young agent (L2.1), the economy-wide resource constraint (L2.2), and* *the macroeconomic production function (L2.3) as follows:*

$$\mathbb{E}\Lambda^y \equiv U(C^y) + \frac{1-\pi}{1+\rho} U(C^o), \tag{L2.1}$$

$$f(k) - (\delta + n)k = C^y + \frac{1-\pi}{1+n} C^o + g, \tag{L2.2}$$

$$f(k) = \Omega_0 k^{\alpha+\eta}. \tag{L2.3}$$

The social planner chooses non-negative values for C^y, C^o, k, and g in order to maximize $\mathbb{E}\Lambda^y$ subject to the constraints (L2.2)–(L2.3). In addition to satisfying the constraints, the first-best social optimum has the following features:

$$\frac{U'(\tilde{C}^y)}{U'(\tilde{C}^o)} = \frac{1+n}{1+\rho}, \tag{S1}$$

$$f'(\tilde{k}) = n + \delta, \tag{S2}$$

$$\tilde{g} = 0. \tag{S3}$$

Proof: See Heijdra *et al.* (2010b, Appendix F). □

Using the terminology of Samuelson (1968), we refer to requirement (S1) of the FBSO as the Biological-Interest-Rate Golden Rule (BGR), and to requirement (S2) as the Production Golden Rule (PGR). Of course, requirement (S3) just states that the social planner does not waste valuable resources.

Armed with Lemma 5.2 we can investigate the efficiency properties of the market economy. In the decentralized equilibrium for the WE scenario the steady-state equilibrium satisfies the resource constraint (L2.2) as well as the following conditions:

$$\frac{U'(\hat{C}^y)}{U'(\hat{C}^o)} = \frac{(1-\pi)(1+\hat{r})}{1+\rho}, \tag{W1}$$

$$\frac{\alpha}{\alpha+\eta} f'(\hat{k}) = \hat{r} + \delta, \tag{W2}$$

$$\hat{g} = \pi (1+\hat{r}) \hat{k}. \tag{W3}$$

Comparing (W1)–(W3) to (S1)–(S3) we find that the WE equilibrium features three distortions. First, the government engages in wasteful expenditure ($\hat{g} > \tilde{g} = 0$). Second, the death probability affects the consumption Euler equation in the decentralized equilibrium i.e. π features in (W1) but not in (S1). There is a missing market in that agents cannot insure against longevity risk. Third, if η is strictly positive the decentralized economy underinvests in physical capital because the capital externality is not internalized by individual agents.

We can rewrite the welfare effect on future steady-state generations – given in (5.30) – as follows:

$$\frac{d\mathbb{E}\Lambda^{y}_{t+\infty}(z_1)}{dz_1} = U'(\hat{C}^y)\frac{\pi(1+n)\hat{k}}{1-\pi}[1-\Theta], \tag{5.31}$$

where Θ is defined as:

$$\Theta \equiv \left[\frac{\eta}{\alpha(1-\alpha-\eta)} + \frac{\hat{r}-n}{1+\hat{r}}\right]\frac{1+\hat{r}}{1+n}\frac{\frac{\hat{r}+\delta}{1+\hat{r}}\Phi(\hat{k})}{1-(1-\sigma)\frac{\hat{r}+\delta}{1+\hat{r}}\Phi(\hat{k})} \geq 0. \tag{5.32}$$

In combination with Lemma 5.2, the expressions in (5.31)–(5.32) can be used to build intuition on the long-run welfare effect of the policy switch from WE to TO. In adopting the TO scenario wasteful government expenditure is eliminated which implies that one distortion is removed, i.e. (S3) holds for the TO case and $\hat{g}^{TO} = \tilde{g} = 0$. If there were no capital externality ($\eta = 0$) *and* the steady-state interest rate would equal the rate of population growth ($\hat{r}^{TO} = n$) then (S2) would also hold under the TO case, i.e. $\hat{k}^{TO} = \tilde{k}$. The only distortion that would remain is the one resulting from the missing insurance market, i.e. $(1-\pi)\left(1+\hat{r}^{TO}\right) < 1+n$. For $\hat{r} = n$ and $\eta = 0$ we find from (5.32) that $\Theta = 0$ and from (5.31) that the long-run welfare effect is strictly positive. The switch from WE to TO benefits all generations to the same extent in this hypothetical case because waste is eliminated, there is no transitional dynamics in the capital stock (and thus in factor prices), and the additional resources lead to an equiproportionate increase in youth and old-age consumption.

Matters are much more complicated if we adopt Assumption 5.1. For $\hat{r} > n$ it follows from (5.32) that Θ is strictly positive and, ceteris paribus $\hat{r}$ and $\hat{k}$, increasing in the externality parameter η. If $\eta = 0$ then WE and TO share two distortions, namely the missing insurance market and the violation of the BGR. It is a straightforward application of the theory of the second best (Lipsey and Lancaster, 1957) that the welfare ranking between WE and TO is ambiguous in that case. In Table 3(a) we compute Θ for several values of the intertemporal substitution elasticity. Interestingly, Θ is strictly larger than unity for all but the most extreme values of σ. And for a relatively small capital externality (Table 3(b) with $\eta = \frac{1}{10}$) the same conclusion holds for *all* admissible values of σ!

In a plausibly parameterized dynamically efficient economy ($\hat{r} > n$), the switch from WE to TO is welfare decreasing because it induces a decrease in the capital intensity and an increase in the interest rate in the long run. Hence, the policy switch moves the economy further away from the FBSO.

Transfers to the young

We consider the welfare effects of a switch from the steady state of the WE case to the TY scenario and let $\hat{C}^o$, $\hat{C}^y$, $\hat{r}$, $\hat{w}$, and $\hat{k}$ denote the steady-state values associated with WE. In the TY scenario the shock-time old do not receive any additional resources, i.e. $d\mathbb{E}\Lambda_{t-1}^{y}(z_2) \,/\, dz_2 = 0$. The welfare effect on the young at the time of the policy switch is given by:

$$\frac{d\mathbb{E}\Lambda_t^y(z_2)}{dz_2} = U'(\hat{C}^y)(1+n)\hat{k}\left[\pi\frac{1+\hat{r}}{1+n} + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_2}\right] > 0, \tag{5.33}$$

where the first term in square brackets is the direct effect and the second term is the general equilibrium effect. The direct effect is positive but the general equilibrium effect is negative because the policy switch boosts capital accumulation which leads to a reduction in the future interest rate. It is not difficult to show, however, that the direct effect is dominant so that welfare rises at impact. In terms of Figure 3(b) the initial budget line passes through point E_0, the lump-sum transfer shifts the line in a

Table 5.3. Value of Θ

$\hat{r} > n$

	(a) $\eta = 0$	(b) $\eta = \frac{1}{10}$	(c) $\eta = \frac{1}{3}$
$\sigma = \frac{1}{2}$	3.29	5.93	17.72
$\sigma = 1$	1.89	3.41	10.19
$\sigma = \frac{3}{2}$	1.33	2.39	7.15
$\sigma = \bar{\sigma}$	0.85	1.41	3.07

parallel fashion to the right, and the decrease in the future interest rate rotates it in a counter-clockwise fashion around point A. The direct effect consists of the move from E_0 to E' and the general equilibrium effect is the move from E' to E_1.

The change in welfare of the future steady-state generations can be written as:

$$\frac{d\mathbb{E}\Lambda^{y}_{t+\infty}(z_2)}{dz_2} = U'(\hat{C}^y)\left[\pi(1+\hat{r})\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_2}\right] > \frac{d\mathbb{E}\Lambda^{y}_{t}(z_2)}{dz_2} > 0, \tag{5.34}$$

where we have used Lemma 5.1 ($\Delta > 0$) and note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{TY}$. Both terms in square brackets are positive so that welfare ambiguously rises in the long run. Indeed, the general equilibrium effect ensures that future generations gain even more than the shock-time generation. The quantitative effects in columns (c), (g), and (k) of Table 2 confirm that, regardless of the magnitude of the intertemporal substitution elasticity, expected lifetime utility increases dramatically as a result of the policy switch. In terms of Figure 3(b), the budget line shifts further to the right in the long run both because the wage increases and transfers are boosted. The decreased interest rate further rotates the budget line but this effect is not large enough to lead to a reduction in the choice set for future generations. Figures 2(b), (f), and (j) illustrate the transition paths of expected lifetime utility for different values of the intertemporal substitution elasticity. Welfare rises monotonically.

In order to develop the economic intuition behind the strong steady-state welfare gain, we rewrite (5.34) as follows:

$$\frac{d\mathbb{E}\Lambda^{y}_{t+\infty}(z_2)}{dz_2} = U'(\hat{C}^y)\frac{\pi(1+n)\hat{k}}{1-\pi}\left[1+\Theta\frac{1-\Phi(\hat{k})}{\Phi(\hat{k})}\right] > 0, \tag{5.35}$$

where Θ is defined in (5.32) above. The switch from WE to TY is welfare increasing because it induces an increase in the capital intensity and a decrease in the interest rate in the long run, i.e. the policy switch moves the economy closer to the FBSO.

5.4 Tragedy of annuitization

In this section we step away from the assumption that the government redistributes accidental bequests or wastes them completely. Instead we analyze the introduction of a private annuity market. An annuity is a financial asset which pays a given return contingent upon survival of the annuitant to the second period of life. If the annuitant dies prematurely then his assets accrue to the annuity firm. Let r^A_{t+1} denote the net rate of return on annuities. Assuming perfect competition among annuity firms, the zero-profit condition is given by $1 + r_{t+1} = (1 - \pi)(1 + r^A_{t+1})$ which implies:

$$1 + r^A_{t+1} = \frac{1 + r_{t+1}}{1 - \pi}. \tag{5.36}$$

It follows that $1 + r^A_{t+1} > 1 + r_{t+1}$, i.e. the return on annuities exceeds the return on regular assets. Hence, in the absence of a bequest motive, it is optimal for the agent to fully annuitize his financial wealth. This confirms findings by *inter alia* Yaari (1965) and Davidoff *et al.* (2005). Under full annuitization agents will no longer leave accidental bequests. In terms of Table 1, the government budget constraint (T1.7) becomes redundant. Savings S_t are replaced one-for-one by annuity holdings A_t, so that (T1.1)-(T1.3) become:

$$C^y_t = \Phi\left(r^A_{t+1}\right) w_t, \tag{T1.1$'$}$$

$$\frac{C^o_{t+1}}{1 + r^A_{t+1}} = \left[1 - \Phi\left(r^A_{t+1}\right)\right] w_t, \tag{T1.2$'$}$$

$$A_t = \left[1 - \Phi\left(r^A_{t+1}\right)\right] w_t. \tag{T1.3$'$}$$

Furthermore, the fundamental difference equation for the capital intensity (T1.8) is replaced by:

$$(1 + n)\, k_{t+1} = \left[1 - \Phi\left(r^A_{t+1}\right)\right] w_t. \tag{T1.8$'$}$$

In the remainder of this section we study the allocation and welfare effects of opening up a perfect annuity (PA) market at time t. We first study the case for which the initial scenario is WE, i.e. the switch is from WE to PA and the initial capital stock features $k_t = \hat{k}^{WE}$. Next we study the case in which the switch is from the TY scenario to perfect annuities. In this case the initial capital stock satisfies $k_t = \hat{k}^{TY}$.

5.4.1 From wasteful expenditure to perfect annuities

Using (5.36) and (T1.5)–(T1.6) in (T1.8′), the fundamental difference equation can be rewritten as follows:

$$[\Psi\left(k_{t+1},z_3\right)\equiv]\ \frac{k_{t+1}}{1-\Phi\left(k_{t+1},z_3\right)}=\Gamma\left(k_t\right), \tag{5.37}$$

where $\Gamma\left(k_t\right)$ is defined in (5.19) above, $\Phi\left(k,z_3\right)$ is given by:

$$\Phi\left(k,z_3\right)\equiv\left[1+\left(1-z_3\pi\right)^{1-\sigma}\left(\frac{1-\pi}{1+\rho}\right)^{\sigma}\left(1-\delta+\alpha\Omega_0 k^{\alpha+\eta-1}\right)^{\sigma-1}\right]^{-1}, \tag{5.38}$$

and z_3 is a perturbation parameter ($0\leq z_3\leq 1$). The partial derivative of $\Psi\left(k_{t+1},z_3\right)$ with respect to the capital intensity is positive, $\Psi_k>0$, but the partial derivative for the perturbation parameter depends on the magnitude of the intertemporal substitution elasticity:

$$\Psi_{z_3}\lesseqgtr 0\quad\Leftrightarrow\quad\sigma\gtreqless 1. \tag{5.39}$$

We provide the following proposition.

Proposition 5.4. *[Existence and stability of the PA model] Consider the PA model as given in (5.37)–(5.38) and adopt Assumption 5.2. The following properties can be established:*

(i) The model has two steady-state solutions; the trivial one features $k_{t+1}=k_t=0$, and the economically relevant satisfies $k_{t+1}=k_t=\hat{k}^{PA}$, where $\hat{k}^{PA}$ is the solution to:

$$\frac{\hat{k}^{PA}}{1-\Phi(\hat{k}^{PA},1)}=\frac{\left(1-\alpha\right)\Omega_0(\hat{k}^{PA})^{\alpha+\eta}}{1+n}.$$

(ii) The trivial steady-state solution is unstable whilst the non-trivial solution is stable:

$$0<\frac{dk_{t+1}}{dk_t}<1,\qquad \textit{for } k_{t+1}=k_t=\hat{k}^{PA}.$$

For any given positive initial value the capital intensity converges monotonically to $\hat{k}^{PA}$.

(iii) The steady-state capital intensity satisfies the following inequality:

$$\hat{k}^{PA}\lesseqgtr\hat{k}^{WE}\quad\Leftrightarrow\quad\sigma\lesseqgtr 1$$

Proof: See Heijdra *et al.* (2010b, Appendix D). □

In the benchmark case the intertemporal substitution elasticity is equal to unity, so that it follows from (5.39) that the opening up of annuity markets has no effect on the fundamental difference equation (5.37). There is no transitional dynamics and the economy with perfect annuities features the same steady-state capital intensity as under the WE scenario, i.e. $k_t = \hat{k}^{PA} = \hat{k}^{WE}$ for all t. In terms of Figure 4(a), the initial equilibrium is at point E_0. Full annuitization rotates the budget line in a clockwise fashion and the new equilibrium is at point E_∞ which lies directly above E_1 (since $\sigma = 1$). The additional resources resulting from annuitization are thus shifted entirely to old age.

Figure 2(d) and Table 2(d) confirm that old-age consumption is significantly higher following the policy shock. Note also from Figure 2(d) that the switch from WE to PA is quite different from the switch from WE to TO even though both constitute risk sharing among old agents. In the latter case the anticipated transfers in old age lead to reduced saving during youth which ultimately results in capital crowding out. In contrast, in the former case the savings rate is unaffected by the policy change.

Since transfers are absent both before and after the opening up of annuity markets, the shock-time old are unaffected by this event, i.e. $d\mathbb{E}\Lambda_{t-1}^{y}(z_3)/dz_3 = 0$. The welfare effect on the young at the time of the policy switch is given by:

$$\frac{d\mathbb{E}\Lambda_t^y(z_3)}{dz_3} = U'(\hat{C}^y)(1+n)\hat{k}\left[\pi + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_3}\right] > 0, \tag{5.40}$$

where the first term in square brackets is the direct effect and the second term is the general equilibrium effect. In the special case with $\sigma = 1$ and $k_t = \hat{k}^{PA}$ the latter effect is absent. It is easy to show that for all admissible values of σ welfare unambiguously rises for all post-shock generations – see also Table 2(d) and Figures 2(b), (f), and (j).

The long-run welfare effect is given by:

$$\frac{d\mathbb{E}\Lambda_{t+\infty}^y(z_3)}{dz_3} = U'(\hat{C}^y)\left[\pi(1+n)\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_3}\right] \gtreqless 0, \tag{5.41}$$

where we have used Lemma 5.1 ($\Delta > 0$) and note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{PA}$. The second term in square brackets represents the general equilibrium effect on factor prices. Of course, for $\sigma = 1$ these effects are absent and the impact and long-run effects coincide.

Empirical evidence, however, suggests that σ falls well short of unity. It follows readily from (5.37) and (5.39) that for $\sigma < 1$ the impact and long-run effects on the capital intensity of the opening up of annuity markets are both negative:

$$\left.\frac{dk_{t+1}}{dz_3}\right|_{k_t=\hat{k}^{WE}} = -\frac{\Psi_{z_3}}{\Psi_k} < 0, \qquad \left.\frac{dk_{t+\infty}}{dz_3}\right|_{k_t=\hat{k}^{WE}} = -\frac{\Psi_{z_3}}{\Psi_k - \Gamma'} < 0, \tag{5.42}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{PA}$. Equation (5.40) shows that welfare of the shock-time young increases both because of the direct effect and because of the increase in the future interest rate. In the long run, however, capital crowding out results in a reduction in wages which shrinks the choice set and reduces welfare for future generations. Figures 2(e)–(h) depict the transition paths and Panel B of Table 2 provides quantitative evidence for the case with $\sigma = \frac{1}{2}$. As the comparison between columns (e) and (h) of Table 2 reveals, capital crowding out is so strong that steady-state welfare is lower under perfect annuities than it is under the WE scenario! This is the first instance of a phenomenon which we call "the tragedy of annuitization." Even though it is individually advantageous to make use of annuity products if they are available, their long-run general equilibrium effects lead to a reduction in welfare of future generations.

The intuition behind the tragedy is not hard to come by. In the PA case the decentralized steady-state equilibrium is characterized by the resource constraint (L2.2) in Lemma 5.2 as well as the following conditions:

$$\frac{U'(\hat{C}^y)}{U'(\hat{C}^o)} = \frac{(1-\pi)\left(1+\hat{r}^A\right)}{1+\rho} = \frac{1+\hat{r}}{1+\rho}, \tag{P1}$$

$$\frac{\alpha}{\alpha+\eta} f'(\hat{k}) = \hat{r} + \delta, \tag{P2}$$

$$\hat{g} = 0. \tag{P3}$$

The PA equilibrium removes two of the distortions plaguing the WE equilibrium. First, the availability of annuities eliminates the missing-market distortion, i.e. π does not feature in (P1) whereas it does in (W1). Second, there are no wasteful government expenditures. Indeed, in the absence of the capital externality ($\eta = 0$) and if $\hat{r} = n$

then the PA equilibrium decentralizes the FBSO – compare (S1)–(S3) to (P1)–(P3). But starting from a dynamically efficient economy ($\hat{r} > n$) featuring a plausible value of the intertemporal substitution elasticity ($\sigma = \frac{1}{2}$), the switch from WE to PA is welfare decreasing because it induces capital crowding out and an increase in the interest rate in the long run. Hence, the policy switch moves the economy further away from the FBSO.

5.4.2 From transfers to the young to perfect annuities

We return to the benchmark case (with $\sigma = 1$) and assume that annuity markets are opened up with the economy located in the steady-state equilibrium of the TY scenario, i.e. $k_t = \hat{k}^{TY}$ initially. A policy switch from the TY case to the PA scenario now involves two distinct changes. On the one hand, the availability of annuities boosts the rate at which the young can save. On the other hand, full annuitization implies that accidental bequests are absent so that the transfers to the *future* young are eliminated, i.e. $Z^y_{t+\tau} = 0$ for $\tau = 1, 2, \ldots$. The combined effect of these shocks can be studied with the aid of the following fundamental difference equations:

$$\Psi\left(k_{t+1}, z_3\right) = \Gamma\left(k_t, 1\right), \quad \Psi\left(k_{t+\tau+1}, z_3\right) = \Gamma\left(k_{t+\tau}\right), \quad \tau = 2, 3, \ldots, \tag{5.43}$$

where $\Gamma\left(k_t\right)$, $\Gamma\left(k_t, 1\right)$, and $\Psi\left(k_{t+1}, z_3\right)$ are defined in, respectively, (5.19), (5.23) and (5.37) above. At time t there is a permanent switch from $z_3 = 0$ to $z_3 = 1$. From $t+1$ onwards transfers are absent and the second expression in (5.43) describes the dynamic law of motion. The resulting difference equations are solved using $k_t = \hat{k}^{TY}$ as the initial condition.

Since $\sigma = 1$ the marginal propensity to save out of current resources is constant. The shock-time young still receive transfers. It follows that there is no effect on saving, i.e. $k_{t+1} = \hat{k}^{TY}$. Of course, the young from period $t+1$ onward no longer receive transfers and these generations will reduce their saving. Over time the economy monotonically converges to $\hat{k}^{PA}$ which is strictly less than $\hat{k}^{TY}$ (since, for $\sigma = 1$, $\hat{k}^{PA} = \hat{k}^{WE}$ and $\hat{k}^{TY} > \hat{k}^{WE}$ by Proposition 5.3(iv)). Using (5.43) we find the impact and long-run effects of the policy change on the capital intensity:

$$\left.\frac{dk_{t+1}}{dz_3}\right|_{k_t=\hat{k}^{TY}} = -\frac{\Psi_{z_3}}{\Psi_k}, \qquad \left.\frac{dk_{t+\infty}}{dz_3}\right|_{k_t=\hat{k}^{TY}} = -\frac{\Psi_{z_3}+\Gamma_{z_2}}{\Psi_k-\Gamma'}, \tag{5.44}$$

where $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{PA}$. Recall that $\Psi_k > 0$, $\Gamma' > 0$, $\Gamma_{z_2} > 0$ and $\Psi_{z_3} \lesseqqgtr 0 \Leftrightarrow \sigma \gtreqqless 1$. It follows that there is capital crowding out both at impact and in the long run for realistic values of σ (i.e., $\sigma < 1$) since Ψ_{z_3} is positive in that case.

The key effects can be explained with the aid of Figure 4(b). The initial steady state is at E_0 and income during youth is equal to $\hat{w}^{TY} + \hat{Z}^y$. At impact the transfers are predetermined, but the interest rate at which the young save increases, i.e. the budget line rotates in a clockwise direction. The new equilibrium is at point E_1 which lies directly above point E_0 (since $\sigma = 1$). In the long run, transfers are eliminated, capital is crowded out, the interest rate rises and the wage rate falls. The long-run budget constraint passes through E_∞ which is the new steady-state equilibrium.

We visualize the transitional dynamics (for the case with $\sigma = 1$) in Panel A of Figure 5. The quantitative effects are summarized in Table 2(d). Figure 5(a) confirms the strong crowding-out effect on the capital intensity. Youth consumption of all but the shock-time young falls as a result of the elimination of transfers (panel (c)) and old-age consumption of survivors increases due to the higher return on savings (panel (d)). Comparing columns (c) and (d) in Table 2 we find that long-run output per worker falls by more than five percent.

Since the old do not get any transfers both before and after the opening up of an annuity market and they no longer save, the shock-time old are unaffected by this event, i.e. $d\mathbb{E}\Lambda^y_{t-1}(z_3)/dz_3 = 0$. The welfare effect on the young at the time of the policy switch is given by:

$$\frac{d\mathbb{E}\Lambda^y_t(z_3)}{dz_3} = U'(\hat{C}^y)(1+n)\,\hat{k}\left[\pi + \frac{1}{1+\hat{r}}\frac{dr_{t+1}}{dz_3}\right] > 0. \tag{5.45}$$

The shock-time young benefit for all admissible values of σ, i.e. regardless of whether next period's capital intensity falls ($\sigma < 1$) or rises ($\sigma > 1$). To this generation the benefits of annuitization are clear and simple.

Matters are not so clear-cut for future generations. Indeed, the long-run welfare effect is equal to:

$$\frac{d\mathbb{E}\Lambda^{y}_{t+\infty}(z_3)}{dz_3} = U'(\hat{C}^y)\left[-\pi(\hat{r}-n)\hat{k} + \Delta\frac{dk_{t+\infty}}{dz_3}\right] \lesseqgtr 0. \tag{5.46}$$

where we have used Lemma 5.1 ($\Delta > 0$) and note that $\lim_{\tau\to\infty} k_{t+\tau} = \hat{k}^{PA}$. The first term in square brackets is negative in a dynamically efficient economy but the sign of the second term depends on the strength of the intertemporal substitution effect. For the empirically relevant case, however, we have $0 < \sigma < 1$, capital is crowded out in the long run, and long-run welfare unambiguously falls.[11]

Figure 5(b) shows (for $\sigma = 1$) that lifetime welfare is reduced for all future generations if a private annuity market is opened up. Only the shock-time young benefit from annuitization. Effectively, private annuities redistribute assets from deceased to surviving elderly in an actuarially fair way whereas transferring unintended bequests to the young constitutes an intergenerational transfer. This intergenerational transfer induces beneficial savings effects, which, in the end, lead to higher welfare. This is the second example of a *tragedy of annuitization*. Even though it is individually rational to fully annuitize, this is not optimal from a social point of view. If all agents invest their financial wealth in the annuity market then the resulting long-run equilibrium leaves everyone worse off compared to the case where annuities are absent and accidental bequests are redistributed to the young.

5.4.3 Discussion

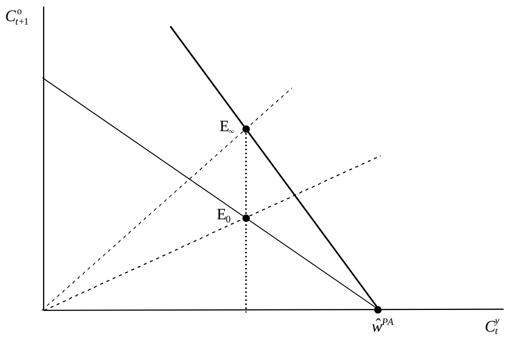
In the previous subsections we have seen two instances of the tragedy of annuitization. The first (from WE to PA) can be considered the *strong* version and the second (from TY to PA) the *weak* version. The remaining question that must be answered is whether or not the tragedy is inescapable. Does the introduction of a perfect annuity market always make future generations worse off?

To answer this question we start by noting that in Table 2 steady-state welfare is lowest for all scenarios considered in the case where accidental bequests are trans-

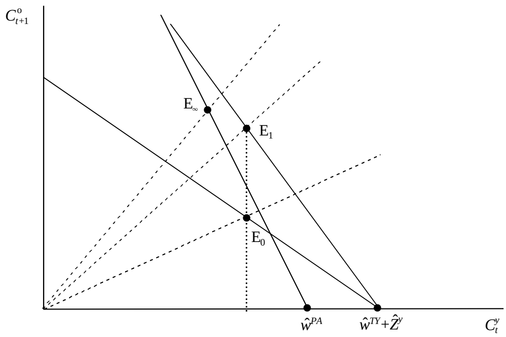
[11] Indeed, the results in Table 2 confirm that the same conclusion holds for $\sigma = \frac{3}{2}$ – compare columns (j) and (l). Of course in that case the capital intensity rises somewhat so that the welfare loss from the switch from TY to PA is smaller.

Figure 5.4. Private annuities in the exogenous growth model

(a) From wasteful expenditure to perfect annuities ($\sigma = 1$)



(b) From transfers to the young to perfect annuities ($\sigma = 1$)



(c) From transfers to the old to perfect annuities ($\sigma = 1$)

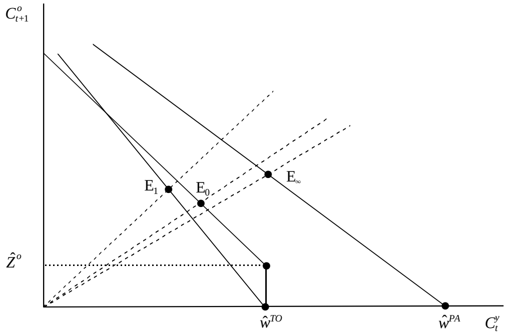
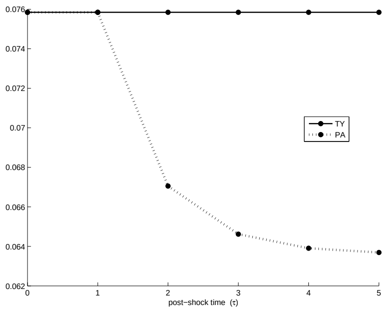


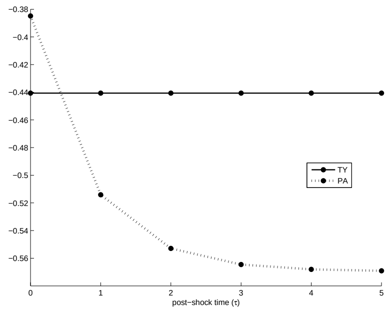
Figure 5.5. Transition from transfers to annuities in the exogenous growth model

Panel A: from TY to PA ($\sigma = 1$)

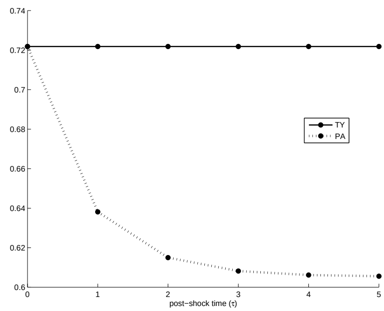
(a) capital intensity ($k_{t+\tau}$)



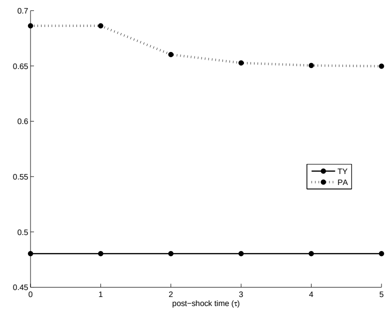
(b) expected lifetime utility ($\mathbb{E}\Lambda^{y}_{t+\tau}$)



(c) youth consumption ($C^{y}_{t+\tau}$)



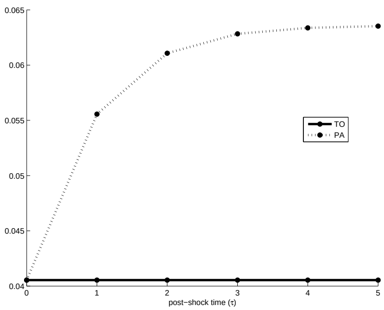
(d) old-age consumption ($C^{o}_{t+\tau}$)



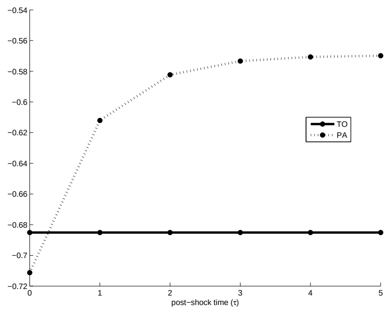
(Figure 5, continued)

Panel B: from TO to PA ($\sigma = 1$)

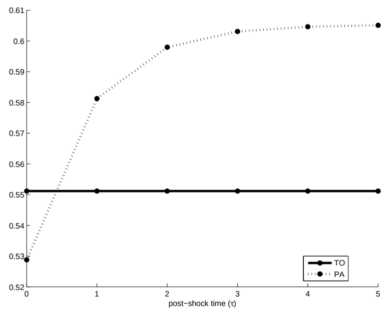
(e) capital intensity ($k_{t+\tau}$)



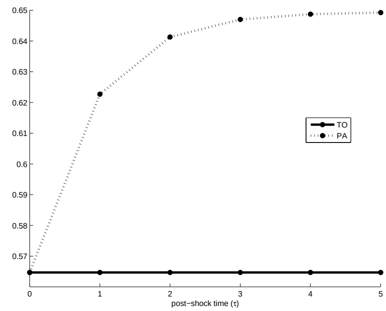
(f) expected lifetime utility ($\mathbb{E}\Lambda^{y}_{t+\tau}$)



(g) youth consumption ($C^{y}_{t+\tau}$)



(h) old-age consumption ($C^{o}_{t+\tau}$)



ferred to the old (the TO scenario). If the switch from TO to PA would still give rise to the tragedy then this would be an even stronger version than the one resulting from the change from WE to PA. It turns out, however, that the tragedy does not arise when annuity markets are opened under the TO scenario.

Formally, the switch from TO to PA again involves two distinct changes. First, full annuitization implies that accidental bequests are absent so that the transfers to all but the shock-time old are eliminated, i.e. $Z^o_{t+\tau} = 0$ for $\tau = 1, 2, \ldots$. Second, the availability of annuities boosts the rate at which the young can save. The combined effect of these shocks can be studied with the aid of (5.37). At time t there is a permanent change from $z_3 = 0$ to $z_3 = 1$ and (5.37) is solved using $k_t = \hat{k}^{TO}$ as the initial condition. Since $\sigma = 1$ in the benchmark case, the marginal propensity to save out of current resources is constant. The elimination of old-age transfers then immediately leads to an increase in saving by the shock-time young, i.e. $k_{t+1} > \hat{k}^{TO}$. Over time the economy monotonically converges to $\hat{k}^{PA}$ which exceeds $\hat{k}^{TO}$ (since, for $\sigma = 1$, $\hat{k}^{PA} = \hat{k}^{WE}$ and $\hat{k}^{TO} < \hat{k}^{WE}$ by Proposition 5.2(iii)).

In the interest of brevity we restrict attention to the key features of the shock. In Figure 4(c) the initial steady state is at E_0, and non-asset income during youth and old-age is, respectively $\hat{w}^{TO}$ and $\hat{Z}^o$. At impact *future* transfers to the shock-time young and all generations thereafter are eliminated and the rate at which the young save increases, i.e. the budget line shifts down and becomes steeper. The new equilibrium is at point E_1. In the long run, the capital intensity increases further, the interest rate falls and the wage rate increases. The long-run budget constraint passes through E_∞ which is the steady-state equilibrium.

We visualize the transitional dynamics (for the case with $\sigma = 1$) in Panel B of Figure 5 and summarize the quantitative results in Table 2(d). Figure 5(e) confirms the strong expansionary effect on the capital intensity. Youth consumption falls at impact as a result of the elimination of old-age transfers (panel (g)) but rises strongly thereafter. Old-age consumption of survivors increases monotonically as a result of the expansion in the choice set made possible by strong capital accumulation (panel (h)). Comparing columns (b) and (d) in Table 2 we find that long-run output per worker increases by almost fifteen percent. Figure 5(f) shows the welfare effect on shock-time and future newborns. Interestingly, the shock-time young are worse off as a result of the introduction of annuity products. For these agents the increase in old-age consumption is insufficiently large to offset the strong decrease in youth consumption. All future newborns, however, are better off as a result of annuitization opportunities.

In Panels B and C of Table 2 we present some steady-state evidence for different values of σ. We find that PA always welfare dominates TO in the long run, regardless of whether the intertemporal substitution effect is weak ($\sigma = \frac{1}{2}$ in Panel B) or strong ($\sigma = \frac{3}{2}$ in Panel C).

The findings in this subsection bear a strong resemblance to the literature on the reform of PAYG pensions. In a dynamically efficient economy, a PAYG system is Pareto efficient. A pension reform in the direction of a fully funded system increases welfare of steady-state generations but harms the shock-time old and possibly the young generations born close to the time of the reform. The scenario considered here differs from the pension reform case because the shock is not policy induced but results from the emergence of a new longevity insurance market.

5.5 The endogenous growth model

In this section we briefly consider the knife-edge case featuring $\eta = 1 - \alpha$. The model then exhibits growth which is driven endogenously by the rate of capital accumulation. We can solve (5.18) for the equilibrium growth rate:

$$(1+n)(1+\gamma) = [1-\Phi(\bar{r})]\left[(1-\alpha)\,\Omega_0 + \frac{Z_t^y}{k_t}\right] - \frac{\Phi(\bar{r})}{1+\bar{r}}\frac{Z_{t+1}^o}{k_t}, \tag{5.47}$$

where $\gamma \equiv k_{t+1}/k_t - 1$ is the (time-invariant) equilibrium growth rate and we have used the fact that the interest rate is constant in this scenario such that $r_t = \bar{r} \equiv \alpha\Omega_0 - \delta$ for all t. Using the expressions in (5.47) we can derive the equilibrium growth rates under the three revenue recycling schemes and after the introduction of a private annuity market.

(WE) If the government uses the proceeds from the accidental bequests for wasteful government expenditures the growth rate becomes:

$$1+\gamma^{WE} = \frac{1-\Phi\left(\bar{r}\right)}{1+n}\left(1-\alpha\right)\Omega_0. \tag{5.48a}$$

(TY) If instead the proceeds are redistributed to the young we find:

$$1+\gamma^{TY} = \frac{1-\Phi\left(\bar{r}\right)}{1+n}\left[\left(1-\alpha\right)\Omega_0+\pi\left(1+\bar{r}\right)\right]. \tag{5.48b}$$

(TO) If the accidental bequests go the elderly then the growth rate is given by

$$1+\gamma^{TO} = \frac{1+\gamma^{WE}}{1+\Phi\left(\bar{r}\right)\frac{\pi}{1-\pi}}. \tag{5.48c}$$

(PA) Finally, if a private annuity market is introduced we have:

$$1+\gamma^{PA} = \frac{1-\Phi(\bar{r}^{A})}{1+n}\left(1-\alpha\right)\Omega_0. \tag{5.48d}$$

Straightforward inspection of the growth rates reveals that $\gamma^{TY} > \gamma^{WE} > \gamma^{TO}$ for all admissible values of σ. Hence, in terms of growth, it is better to give the accidental bequests to the young than to use them for wasteful expenditures, yet it is better to let the accidental bequests go to waste than to give them to the elderly.

Comparison with the private annuities scenario is more subtle. The introduction of private annuities increases the rate against which individuals save. The savings response of consumers, and thereby the growth rate in the perfect annuities scenario relative to the various recycling schemes, depends on the value of the intertemporal elasticity of substitution σ. For the benchmark case with $\sigma = 1$ savings are independent of the interest rate and $\gamma^{TY} > \gamma^{PA} = \gamma^{WE} > \gamma^{TO}$. If $0 < \sigma < 1$ the higher interest rate will lead to less savings than in the benchmark scenario so that we get $\gamma^{TY} > \gamma^{WE} > \gamma^{PA} > \gamma^{TO}$. Finally, if $\sigma > 1$ the higher interest rate will lead to more savings which results in $\gamma^{PA} > \gamma^{WE} > \gamma^{TO}$ and, depending on the exact magnitude of σ, $\gamma^{PA} \gtreqless \gamma^{TY}$.

In order to compare consumer welfare across the various scenarios we must recognize the fact that steady-state expected lifetime utility grows at a scenario-dependent rate in an endogenous growth model. To see this, note that if $\eta = 1 - \alpha$ we can write the consumption demand equations (5.5) and (5.6) under scenario i as:

$$C^{y,i}_{t+\tau} \equiv \Phi\left(r^i\right)\theta^i w^i_{t+\tau}, \qquad C^{o,i}_{t+\tau+1} \equiv (1+r^i)\left[1-\Phi\left(r^i\right)\right]\theta^i w^i_{t+\tau}, \tag{5.49}$$

where $r^i = \bar{r}$ for $i \in \{\text{WE,TY,TO}\}$ and $r^i = \bar{r}^A$ for $i = \text{PA}$. The value of the parameter θ^i depends on the specific scenario $i \in \{\text{WE,TY,TO,PA}\}$.[12] Wages grow over time according to the equilibrium growth rate associated with scenario i:

$$w^i_{t+\tau} = \left(1+\gamma^i\right)^\tau w_t. \tag{5.50}$$

Consider an economy that is initially in the WE scenario and features a wage rate at time t equal to w_t. Expected lifetime utility of future newborns under scenario i can then be written as:

$$\widehat{\mathbb{E}\Lambda}^{y,i}_{t+\tau} \equiv \begin{cases} \dfrac{\Phi\left(r^i\right)^{-1/\sigma}\left[\theta^i\left(1+\gamma^i\right)^\tau w_t\right]^{1-1/\sigma} - \dfrac{2+\rho-\pi}{1+\rho}}{1-1/\sigma} & \text{for } \sigma > 0,\ \sigma \neq 1 \\[2ex] \Xi_0 + \dfrac{2+\rho-\pi}{1+\rho}\left[\theta^i\left(1+\gamma^i\right)^\tau w_t\right] + \dfrac{1-\pi}{1+\rho}\ln\left(1+r^i\right) & \text{for } \sigma = 1 \end{cases} \tag{5.51}$$

We call this welfare metric normalized utility. Clearly, $\widehat{\mathbb{E}\Lambda}^{y,i}_{t+\tau}$ depends both on post-shock time τ and on the scenario-dependent (endogenous) value of γ^i. From equation (5.51) we observe that with the introduction of a transfer regime or an annuity market there is both a *level* effect (represented by a change in the θ^i parameter) and a *growth* effect (induced by a change in γ^i). However, over time the growth effect will always dominate the level effect.

In order to quantify the growth and welfare effects we adopt the following approach. For n, π, α, δ, and r we use the same values as for the exogenous growth model (see the text below Proposition 5.1). We calibrate an annual growth rate of one percent in the WE scenario ($\gamma^{WE} = 0.49$) and obtain $\Omega_0 = 15.72$ and $\rho = 1.78$ (or 2.58% annually). The equilibrium growth rate under the various policy schemes is reported

[12] For the three public policy regimes we get $\theta^{WE} = 1$, $\theta^{TY} = \left[1 + \frac{\pi(1+\bar{r})}{(1-\alpha)\Omega_0}\right]$, and $\theta^{TO} = \left[1 + \pi\frac{1+n}{1-\pi}\frac{1+\gamma^{TO}}{(1-\alpha)\Omega_0}\right]$. For private annuities $r^i = \bar{r}^A$ and $\theta^{PA} = 1$.

in Table 4 for different values of σ and the corresponding welfare paths are depicted in Figure 6.

Table 5.4. Annual steady-state growth rates with endogenous growth

$$\eta = 1 - \alpha$$

	(a) $\sigma = \frac{1}{2}$	(b) $\sigma = 1$	(c) $\sigma = \frac{3}{2}$
WE	1.00	1.00	1.00
TO	0.26	0.26	0.26
TY	1.31	1.31	1.31
PA	0.64	1.00	1.35

In line with the exogenous growth model we find that if the economy exhibits endogenous growth and the intertemporal substitution elasticity is in the realistic range ($0 < \sigma \leq 1$) then it is better to transfer the proceeds of accidental bequests to the young than to open up a private annuity market – see Table 4 and Figure 6. In addition we find that for low values of σ it may even be better to waste the accidental bequests than to have a system of private annuities. Hence, both the weak and the strong version of the tragedy of annuitization show up in terms of economic growth rates.

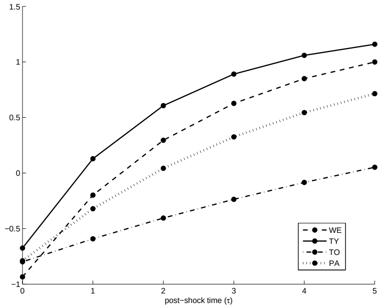
Finally, we find that only if σ is unrealistically high (e.g., $\sigma = \frac{3}{2}$) private annuities slightly outperform transfers to the young in terms of growth – see Table 4(c). However, in terms of welfare, PA only outpaces the TY scenario after three periods (i.e. 120 years) and even then only marginally so – see Figure 6(c).

5.6 Conclusion

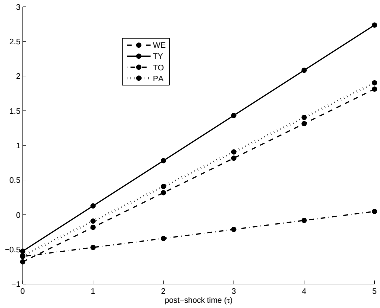
We construct a tractable discrete-time overlapping generations model of a closed economy featuring endogenous capital accumulation. We use this model to study government redistribution and private annuities in general equilibrium. Individuals face longevity risk as there is a positive probability of passing away before the retirement period. With an uncertain life expectancy, non-altruistic agents engage in precautionary saving to avoid running out of assets in old age. While they refrain from leaving intentional bequests to their offspring, they will generally make *unintended* bequests

Figure 5.6. Welfare paths in the endogenous growth model

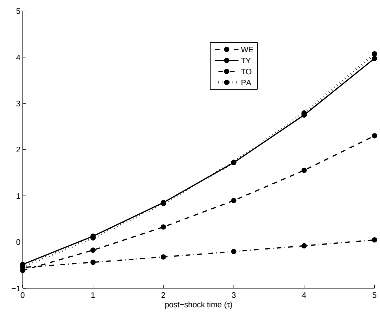
(a) Weak intertemporal substitution effect: $\sigma = \frac{1}{2}$



(b) Benchmark: $\sigma = 1$



(c) Strong intertemporal substitution effect: $\sigma = \frac{3}{2}$



which we assume to flow to the government. Starting from a case in which the government initially wastes these resources, we investigate the effects on allocation and welfare of various revenue recycling schemes. Interestingly, we find non-pathological cases where it is better for long-run welfare to waste accidental bequests than to give them to the elderly. This is because transfers received in old age cause the individual to reduce saving which at the macroeconomic level results in a dramatic fall in the capital intensity and in wages.

Next we study the introduction of a perfectly competitive annuity market offering actuarially fair annuitization products. We demonstrate that there exists a *tragedy of annuitization*: although full annuitization of assets is privately optimal it may not be socially beneficial due to adverse general equilibrium repercussions. For example, if the economy is initially in the equilibrium with accidental bequests flowing to the young, then opening up annuity markets will reduce steady-state welfare regardless of the magnitude of the intertemporal substitution elasticity. Intuitively, private annuities redistribute assets from deceased (unlucky) individuals to surviving (lucky) elderly in an actuarially fair way, whereas transferring unintended bequests to the young constitutes an intergenerational transfer. This intergenerational transfer induces beneficial savings effects, which, in the end, lead to higher welfare.

The existence of the tragedy is the rule rather than the exception. We find an even stronger version which states that revenue wasting dominates perfect annuitization, and we show that it also turns up in an endogenous growth context.

Although the current framework is quite general, an interesting alley for future research is to study how the current model can be generalized further. The most obvious directions in this respect are a more general utility function and/ or production technology. Especially regarding the utility side of the model, a function that allows for gains due to certainty to be included in welfare considerations would add interesting considerations. In addition to a more general framework, an extension to allow for a more active government is of interest. For instance, it would be interesting to consider whether the government can allocate the gains from a tax in such a way that the negative impact of annuities can be counteracted.

This chapter closes our analysis of the role of annuity markets in general equilibrium. In the next, and final, chapter we return to the analysis of the consequences of demographic change for the macroeconomy. However, in contrast to Chapter 4 we do not focus on the moderating role of the pension system but we study how different types of demographic shocks affect the macroeconomy.

Chapter 6

Capital accumulation and the sources of demographic change*

* This chapter is based on Mierau and Turnovsky (2011).

6.1 Introduction

In this chapter we return to the analysis of demographic change initiated in Chapter 4. However, rather than studying the moderating role of the pension system, we use this chapter to study how different types of demographic change affect the aggregate economy. How does a change in the birth rate affect the capital stock? How does a change in the mortality rate affect the capital stock? And what is the impact of a combined mortality and birth rate shock?

To analyse these issues, we construct a model similar to the one in Chapter 2 but it differs on a number of key issues. First of all, we step away from the analysis of an endogenous growth model and focus on an exogenous growth model in which all factor prices are endogenous. This set-up allows us to study how demographic changes affect the aggregate capital stock and gives us a basic idea of the dynamics governing the model. Second, we focus on a model featuring perfect annuity markets instead of imperfect annuity markets. As we saw in Chapter 2, annuity market imperfections, although very prevalent, have a mild impact if proper account is taken of the demographic structure and the redistribution of profits made by the annuity firms. In the same spirit as Chapter 2 we stick with the analysis of an age-dependent mortality rate.

In the first part of the analysis we study the theoretical aspects of the model. We build on the contribution of d'Albis (2007) by highlighting the mechanisms whereby the demographic structure impedes on the macrodynamic equilibrium. This is through the "generational turnover term", which refers to the reduction in aggregate consumption due to the addition of newborn agents having no accumulated assets, together with the departure of agents with accumulated lifetime assets. Different demographic structures share the feature that they impact on the aggregate macrodynamic equilibrium through their effect on the aggregate consumption growth rate. Hence, differences among various demographic structures reduce to differences in the specification of the generational turnover term. By explicitly setting out the underlying dynamic system, we are able to establish that there are in fact two steady-state equilibria, rather than just the one identified in d'Albis (2007) and much of the remaining literature.

The two equilibria contrast sharply in how they are influenced by the demographic structure. In the first equilibrium (the one generally identified in the literature) demographic factors play an important role. They impede on equilibrium per capita consumption directly, through the impact of the mortality function on the discounting of future consumption. In contrast, in the second equilibrium we identify, demographic factors play no direct role, except insofar as they influence the overall population growth rate. The key feature of this equilibrium is that the equilibrium growth rate of consumption just equals the growth rate of population. As a result, the amount of consumption given up by the dying just equals that required to sustain the consumption of the growing population. Accordingly, steady-state consumption is sustainable, independent of the time profile of the underlying mortality function. However, we show that this latter steady-state is unsupported by any underlying dynamic path and is sustainable only in the presence of intergenerational transfer flows, much like the "bubble" steady-state in the Bommier-Lee (2003) model. We thus effectively dismiss it as a relevant equilibrium for the current analysis and focus on the "demographic" equilibrium for the remainder of the analysis.

To enhance our understanding of the dynamics of the model and to prepare for the numerical analysis, we must add more demographic structure, and we do so by adopting the Boucekkine, de la Croix, and Licandro (2002) (BCL) mortality function that we previously employed in Chapters 2 through 4. Using the BCL function we provide an explicit representation of the aggregate macrodynamic system. This turns out to be a highly nonlinear fifth order system involving not only capital and consumption, as in the standard representative agent economy, but also the dynamics of the various elements of the intergenerational turnover term. This model embeds the Blanchard model, the dynamics of which simplify dramatically due to the constant mortality assumption, which carries the implication that both human wealth and the marginal propensity to consume are independent of age. As it stands, the dynamic system cannot be solved explicitly and we focus on the steady-states in a numerical analysis. Naturally, the dynamics (and especially transitional dynamics) remains the obvious next step for future research.

In the numerical simulations we study the steady-state behavior of the model in response to both structural and demographic changes, illustrating their effects on aggregate quantities, as well as on the distributions of consumption and wealth across cohorts. Our numerical results show how the effects of a given increase in the population growth rate contrast sharply – both qualitatively and quantitatively – depending upon whether it occurs through an increase in the birth rate or a decrease in mortality. Whereas in the former case an increase in the population growth rate is associated with a mild decline in the capital stock, in the latter case it leads to a substantial increase in the per capita stock of capital. These differences in turn carry over to other aspects of the aggregate economy.

This contrast echos the results of Heijdra and Lighthart (2006) who study the two different types of demographic shocks in a perpetual youth overlapping generations model. Although the perpetual youth model gives more space to the analytical analysis of the dynamics, the magnitude of the results can be misleading due to the emphasis that the model puts on the elderly who are additionally endowed with too many assets (see also section 2.4.2 and 6.5.1). The contrast between the different types of demographic shocks is also consistent with empirical evidence obtained by Blanchet (1988) and by Kelley and Schmidt (1995). The latter summarize the difference in terms of children, having little accumulated wealth, being "resource users" and working adults with their accumulated capital being "resource creators".[1] Our numerical results also confirm the empirical findings of Bloom, Canning, and Graham (2003) who find that increases in life expectancy leads to higher savings, as well as the consumption patterns obtained by Fair and Dominguez (1991), Attfield and Cannon (2003), and Erlandsen and Nymoen (2008).

As it stands, the current chapter studies mainly theoretical and quantitative issues pertaining to fertility and mortality in the neoclassical framework. However, just as the model in Chapter 2 served as a stepping stone to the analysis of taxation and pensions in Chapters 3 and 4, the current chapter will serve as a stepping stone for the analysis of public policy issues in future research. Naturally, revisiting the topics of taxation and pensions is a natural starting point for further analysis.

The remainder of the chapter is structured as follows. Section 2 lays out the components of the underlying analytical framework, while section 3 describes the corresponding macrodynamic equilibrium and steady state. Section 4 focuses on specific

[1] It is also consistent with the related evidence from cross-country studies of fertility and growth. These have typically found the correlations between economic growth and population growth to be negative for less developed economies, having higher birth rates, and positive for developed economies, with their lower mortality rates (Kelley, 1988).

demographic structures and section 5 performs the numerical simulations. The final section concludes and provides some suggestions for directions in which this research might be extended.

6.2 The analytical framework

The model developed and analyzed in this chapter shares many features with the model developed in Chapter 2. Hence, in what follows we keep the explanation brief and pay attention to the differences between the models rather than the similarities. The most important differences being that in this chapter we focus on perfect annuity markets and that we study an exogenous growth model instead of an endogenous growth model.

6.2.1 Individual household behavior

Discounted expected life-time utility of an individual newborn at time v is:

$$E\Lambda(v) = \int_{v}^{v+\bar{D}} U\left(C(v,t)\right) \cdot e^{-\rho(t-v)-M(t-v)} dt, \tag{6.1a}$$

where $C(v,t)$ denotes the consumption at time t of an individual born at time v, ρ is the pure rate of time preference, $M(t-v) \equiv \int_0^{t-v} \mu(s)\, ds$ is the cumulative mortality rate, $\mu(s)$ is the instantaneous probability of death and $\bar{D}$ is the maximum attainable age. The agent supplies a unit of labor inelastically and is assumed to make his consumption and asset accumulation decisions to maximise his expected utility (6.1a) subject to his budget constraint:

$$A_t(v,t) \equiv \frac{\partial A(v,t)}{\partial t} = \left(r(t) + \mu(t-v)\right) A(v,t) + w(t) - C(v,t), \tag{6.1b}$$

where $A(v,t)$ are real assets held at time t of an individual born at time v, $w(t)$ is the wage rate, and $r(t)$ is the real interest rate (see below). Individuals are born without assets, have no bequest motive, and are not allowed to die indebted. Therefore, $A(v,v) = 0$, and individuals fully annuitize all their assets against the rate of return

$r(t) + \mu(t-v)$.[2]

Defining the present value Hamiltonian for an agent born at time v:

$$H \equiv e^{-\rho(t-v)-M(t-v)} \left\{U\left(C(v,t)\right) + \lambda(v,t)\left[\left(r(t) + \mu(t-v)\right) A(v,t) + w(t) - C(v,t)\right]\right\}$$

and optimizing with respect to $C(v,t)$ and $A(v,t)$, we obtain:

$$U'\left(C(v,t)\right) = \lambda(v,t), \tag{6.2a}$$

$$\rho - \frac{\lambda_t(v,t)}{\lambda(v,t)} = r(t). \tag{6.2b}$$

Equation (6.2a) equates the marginal utility of consumption to the shadow value of financial wealth, while (6.2b) equates the rate of return on consumption, adjusted by the mortality hazard rate, to the rate of return on financial assets. In addition, the agent must satisfy the transversality condition: $A(v, v+D) = 0$.

For analytical convenience we assume an iso-elastic utility function:

$$U\left(C(v,t)\right) = \frac{C(v,t)^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where σ is the intertemporal elasticity of substitution. Combining (6.2a) and (6.2b) enables us to write the Euler equation as:

$$\frac{C_t(v,t)}{C(v,t)} = \sigma\left(r(t) - \rho\right), \tag{6.3}$$

which expresses how the agent's consumption changes with age. In particular, equation (6.3) implies that consumption of all agents grows at a common rate, independent of their age or level of wealth.

Solving (6.3) forward from time t, the agent's consumption at an arbitrary time $\tau > t$ is:

$$C(v,\tau) = C(v,t)e^{\sigma(R(t,\tau)-\rho(\tau-t))}, \tag{6.4}$$

where $R(t,\tau) \equiv \int_t^\tau r(s)\,ds$ is the cumulative interest rate over period (t,τ). To express the agent's consumption in terms of financial resources, we integrate the budget

[2] In this chapter we focus on perfectly functioning annuity markets. See Chapters 2-4 for an analysis of imperfect annuity markets.

constraint (6.1b) forward from time t and impose the transversality condition. This procedure yields the agent's intertemporal budget constraint operative from time t:

$$\begin{aligned} & A(v,t) + e^{R(v,t)+M(t-v)} \int_t^{v+\bar{D}} w(\tau) e^{-R(v,\tau)-M(\tau-v)} d\tau \\ = \; & e^{R(v,t)+M(t-v)} \int_t^{v+\bar{D}} C(v,\tau) e^{-R(v,\tau)-M(\tau-v)} d\tau. \end{aligned} \tag{6.5}$$

Substituting (6.4) into (6.5) we obtain the following expression for $C(v,\tau)$:

$$C(v,t) = \frac{A(v,t) + \int_t^{v+\bar{D}} w(\tau) e^{-R(t,\tau)-(M(\tau-v)-M(t-v))} d\tau}{\int_t^{v+\bar{D}} e^{(\sigma-1)R(t,\tau)-\sigma\rho(\tau-t)-(M(\tau-v)-M(t-v))} d\tau} = \frac{A(v,t)+H(v,t)}{\Delta(v,t)}, \tag{6.6a}$$

where:

$$H(v,t) \equiv \int_t^{v+\bar{D}} w(\tau) e^{-R(t,\tau)-(M(\tau-v)-M(t-v))} d\tau, \tag{6.6b}$$

is discounted future labour income (human wealth) at time t of an individual born at time v, and:

$$\Delta(v,t) \equiv \int_t^{v+\bar{D}} e^{(\sigma-1)R(t,\tau)-\sigma\rho(\tau-t)-(M(\tau-v)-M(t-v))} d\tau, \tag{6.6c}$$

is the inverse of the marginal propensity to consume out of total wealth (i.e. financial wealth, $A(v,t)$, plus human wealth, $H(v,t)$) at age $t-v$. Expressions (6.6b) and (6.6c) show that an increase in the mortality rate leads to a decline in human wealth and an increase in the marginal propensity to consume, as agents will have a shorter expected lifespan over which to accumulate assets and to consume the income they generate. Setting $t = v$ yields the corresponding quantities at birth.

6.2.2 Aggregate household behavior

To obtain aggregate per capita quantities, we sum across cohorts by employing the following generic aggregator function:

$$x(t) \equiv \int_{t-\bar{D}}^t p(v,t) X(v,t) dv = \beta \int_{t-\bar{D}}^t e^{-n(t-v)-M(t-v)} X(v,t) dv, \tag{6.7}$$

where $p(v,t)$ denotes the relative size of the cohort born at time v that is still alive at time t. Taking the time derivative of (6.7), the evolution of $x(t)$ is given by:

$$\dot{x}(t) = \beta X(t,t) + \int_{t-\bar{D}}^{t} p(v,t)\, X_t(v,t)\, dv - nx(t) - \int_{t-\bar{D}}^{t} \mu(t-v)\, p(v,t)\, X(v,t)\, dv, \tag{6.8}$$

where we have used the fact that (see Box 2.2 for details) that $p(t,t) = \beta$, and $p(t-D,t) = 0$.

Thus, aggregate consumption is:

$$c(t) \equiv \int_{t-\bar{D}}^{t} p(v,t)\, C(v,t)\, dv. \tag{6.9}$$

Taking the time derivative of (6.9), the dynamics of per capita consumption is described by:

$$\dot{c}(t) = \frac{dc(t)}{dt} = (\sigma[r(t)-\rho] - n)\, c(t) + \beta C(t,t) - \int_{t-\bar{D}}^{t} \mu(t-v)\, p(v,t)\, C(v,t)\, dv. \tag{6.10}$$

Combining (6.10) with (6.3) we see that:

$$\frac{\dot{c}(t)}{c(t)} = \frac{C_t(v,t)}{C(v,t)} - \frac{\Phi(t)}{c(t)} \tag{6.11a}$$

where:

$$\Phi(t) \equiv \int_{t-\bar{D}}^{t} \mu(t-v)\, p(v,t)\, C(v,t)\, dv - \beta C(t,t) + nc(t) \tag{6.11b}$$

is the "generational turnover term". That is, the reduction in aggregate per capita consumption (below the common consumption growth rate of each cohort) due to the addition of newborn agents with no accumulated assets and the departure of agents with assets. It depends upon: (i) total consumption given up by the dying relative to the average; and (ii) the difference between the consumption of a newborn and the overall average per capita consumption due to growth.

The expression in (6.11b) provides a very general specification that encompasses all of the standard demographic models. With zero population growth, the textbook infinitely-lived representative agent model is obtained by setting $\beta = \mu = 0$ (implying $D \to +\infty$). If there is (disembodied) population growth, we need to take account of the fact that at each instant each newborn is immediately endowed with the average capital stock, part of which he must immediately set aside for the individuals born at the next instant. With the intertemporal elasticity of substitution σ, this reduces the per capita consumption growth rate by $\Phi(t)/c(t) = \sigma n$, so that (6.10) reduces to the familiar aggregate Euler equation $\dot{c}(t) = \sigma(r(t) - \rho - n)c(t)$. For a more realistic demographic process, one in which agents are born and eventually die, we get the more general aggregate Euler equation described in (6.11a). In that case the exact nature of the demographic process will determine the structure of Φ (see below).

Integrating by parts and simplifying, yields:

$$\begin{aligned}\Phi(t) &= -\beta \int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)}[nC(v,t) + C_v(v,t)]dv + nc(t) \\ &= -\beta \int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)} C_v(v,t)\, dv \qquad \text{(6.11b')}\end{aligned}$$

where $C_v(v,t)$ represents the change in consumption across cohorts at a given point in time. Hence, using (6.12) in (6.10) the evolution of aggregate per capita consumption can be written as:

$$\dot{c}(t) = \sigma[r(t) - \rho]c(t) - \beta \int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)} C_v(v,t)\, dv. \tag{6.12}$$

To determine the sign of $\Phi(t)$ we use the fact that at any instant in time, the rate of change of consumption of agents of age $t - v$ is $\dot{C}(v,t) = C_v(v,t) + C_t(v,t)$.[3] Recalling (6.3), and letting $\gamma(v,t) \equiv \dot{C}(v,t)/C(v,t)$ denote the growth rate of consumption this implies:

$$C_v(v,t) = [\gamma(v,t) - \sigma[r(t) - \rho]]\, C(v,t).$$

Thus, a sufficient condition to ensure that $\Phi(t) > 0$ is that the growth rate of consumption with age exceeds the overall growth rate of consumption. In steady-state, $\gamma(v,t) = 0$, implying that $C_v(v,t) = -C_t(v,t) = -\sigma(r(t) - \rho)c(t)$ and we immediately derive $\Phi(t) = \sigma(r(t) - \rho)c(t)$.

Employing (6.7) again, aggregate per capita assets are:

$$a(t) \equiv \int_{t-\bar{D}}^{t} p(v,t)\, A(v,t)\, dv. \tag{6.13}$$

[3] Formally the rate of change of consumption of $t - v$ year old agents is $\lim_{h\to 0} \frac{C(v+h,t+h) - C(v,t)}{h}$.

Taking the time derivative of (6.13) and using (6.1b), per capita asset accumulation is determined by:

$$\begin{aligned} \dot{a}(t) &= \int_{t-\bar{D}}^{t} p(v,t)\left[(r(t)+\mu(t-v))A(v,t)+w(t)-C(v,t)\right]dv \\ &\quad -\int_{t-\bar{D}}^{t}\left[n+\mu(t-v)\right]\cdot p(v,t)A(v,t)\,dv \end{aligned}$$

so that

$$\dot{a}(t) = (r(t)-n)\,a(t)+w(t)-c(t)\,, \tag{6.14}$$

where we have used the fact that $A(t,t)=0$. The per capita rate of asset accumulation differs from the individual rate of asset accumulation, due to the fact that (i) the amount μA is a transfer by insurance companies from those who die to those who remain alive and thus does not add to aggregate wealth; and (ii) account has to be taken of the growing population.

6.2.3 Firms

Output is produced by a representative firm in accordance with the neoclassical production function having constant returns to scale and adhering to the Inada-conditions:

$$\begin{aligned} Y(t) &= F(K(t),L(t))\,,\;\; F_K>0, F_L>0, F_{KK}<0, F_{LL}<0, F_{LK}>0, \qquad (6.15) \\ \lim_{K\to 0} F_K &= \lim_{L\to 0} F_L = \infty \text{ and } \lim_{K\to\infty} F_K = \lim_{L\to\infty} F_L = 0, \end{aligned}$$

where $Y(t)$ is output, $K(t)$ is capital, and $L(t)$ is aggregate labor supply. In per capita terms this may be expressed as:

$$\frac{Y(t)}{L(t)} \equiv y(t) = F\left(\frac{K(t)}{L(t)},1\right) = f(k(t))\,. \tag{6.15'}$$

Assuming that labor and capital are paid their marginal products, the equilibrium wage rate and return to capital are determined by:

$$w(t) = f(k(t)) - f'(k(t))\,k(t)\,, \tag{6.16a}$$

$$r(t) = f'(k(t)) - \delta, \tag{6.16b}$$

where δ is the depreciation rate of capital.

6.3 General Equilibrium

In equilibrium, both the capital and the labor market must clear. Labor market clearance is reflected in the fact that all agents are fully employed so that the total population equals the total labor force. Capital market equilibrium is imposed by setting aggregate assets equal to total capital $A(t) = K(t)$, so that in aggregate per capita terms $a(t) = k(t)$, implying further that $\dot{a}(t) = \dot{k}(t)$.

Substituting the factor pricing relations (6.16) into (6.14) and (6.12) enables us to summarize the dynamics of the macroeconomic equilibrium in the form:

$$\dot{k}(t) = f(k(t)) - c(t) - (\delta + n)\,k(t) \tag{6.17a}$$

$$\dot{c}(t) = \sigma\left(f'(k(t)) - \delta - \rho\right) c(t) - \Phi(t) \tag{6.17b}$$

where

$$\Phi(t) = -\beta \int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)} C_v(v,t)\,dv. \tag{6.17c}$$

This pair of dynamic equations in $\dot{k}$ and $\dot{c}$ will be recognized as being a variant of the standard textbook neoclassical growth model. Equation (6.17a) is the standard aggregate per capita accumulation of capital relationship, where the normalization of individual labor supply at unity implies that that aggregate labor supply is equal to one, while (6.17b) is the aggregate Euler equation, determining the intertemporal allocation of consumption.

The key point to emphasize with regard to expressing the macroeconomic equilibrium in this way is that it highlights how the demographic structure impedes on the economy through the generational turnover term, $\Phi(t)$, and its impact on the aggregate Euler equation. It provides a very general representation in which various specifications of the demographic structure can be embedded. In the case of the pioneering Blanchard (1985) model, and variants such as those developed by Buiter (1988) and Weil (1989), the evolution of (6.17) is very straightforward and the full model can be described by a three dimensional dynamic system; see e.g. Blanchard (1985, p.234) and below.

However, the fact that Φ depends upon how consumption at any instant of time varies across cohorts means that for more general demographic structures its dynamic evolution can be very complex. As we demonstrate in Section 4 below, a more realistic demographic structure leads to a much higher dynamic system, due to the fact that the marginal propensity to consume varies over the life-cycle. In general, in order to characterize the aggregate dynamics and to prevent them from being totally intractable it is necessary to impose some constraints on the demography.[4]

6.3.1 Steady-State

In the steady-state, consumption, asset accumulation, relative cohort size, survival and mortality no longer depend upon calendar time but only on age ($u \equiv t - v$). As a result, with no long-run per capita growth, per capita consumption, $c(t)$, per capita capital stock, $k(t)$, the wage rate, $w(t)$, the return to capital, $r(t)$, and the generational transfer term, $\Phi(t)$, are all constant over time. We shall denote all steady-state quantities by tildes.

Thus, when the aggregate economy is in steady state, consumption grows at the steady rate $\sigma(\tilde{r} - \rho)$ with age, so that the consumption level of an individual of age is equal to:

$$\tilde{C}(u) = \tilde{C}_0 e^{\sigma(\tilde{r}-\rho)u} \tag{6.18}$$

where, setting $t = v$ in (6.6a), consumption at birth, $\tilde{C}_0$ can be expressed as:

$$\tilde{C}_0 = \frac{\tilde{w} \int_0^{\bar{D}} e^{-\tilde{r}u - M(u)} du}{\int_0^{D} e^{-(\tilde{r}(1-\sigma)+\sigma\rho)u - M(u)} du}. \tag{6.19}$$

In the steady-state $p(v,t) = p(u) = \beta e^{-nu-M(u)}$ implying that aggregate consumption per capita is:

$$\tilde{c} \equiv \int_0^{\bar{D}} p(u)\,\tilde{C}(u)\,du = \beta \tilde{C}_0 \int_0^{\bar{D}} e^{(\sigma(\tilde{r}-\rho)-n)u - M(u)} du. \tag{6.20}$$

[4] Having obtained $k(t)$, one can determine the time paths for the return to capital $r(t)$ and the wage rate $w(t)$. Having obtained these one can then derive the dynamics of consumption, savings, and capital accumulation across cohorts.

Defining the function:[5]

$$\Xi(\lambda) \equiv \int_0^{\bar{D}} e^{-\lambda s - M(s)} ds,$$

we can combine (6.18)-(6.20) to express the steady-state per capita consumption, (6.20) as:

$$\tilde{c} = \tilde{w} \frac{\Xi(\tilde{r})}{\Xi(\tilde{r}(1-\sigma)+\sigma\rho)} \frac{\Xi(n-\sigma(\tilde{r}-\rho))}{\Xi(n)}. \tag{6.21}$$

Finally, using the demographic steady-state condition,

$$\frac{1}{\beta} = \int_0^{\bar{D}} e^{-nu - M(u)} du = \Xi(n)$$

we can write:

$$\tilde{c} = \beta\tilde{w} \frac{\Xi(\tilde{r}) \cdot \Xi(n-\sigma(\tilde{r}-\rho))}{\Xi(\tilde{r}(1-\sigma)+\sigma\rho)} \tag{6.21'}$$

as in d'Albis (2007, p.416).

Substituting for the steady-state factor prices, (6.16), the steady-state equilibrium values of per capita consumption, $\tilde{c}$, and capital, $\tilde{k}$, are jointly determined by:

$$\tilde{c} = f(\tilde{k}) - (\delta+n)\tilde{k} \tag{6.22a}$$

$$\tilde{c} = \beta[f(\tilde{k}) - \tilde{k}f'(\tilde{k})] \frac{\Xi(f'(\tilde{k}-\delta)) \cdot \Xi(n-\sigma(f'(\tilde{k})-\delta-\rho))}{\Xi([f'(\tilde{k})-\delta](1-\sigma)+\sigma\rho)} \tag{6.22b}$$

where the demographic characteristics are embedded in the Ξ-function. Letting $s(\tilde{k}) \equiv \tilde{k}f'(\tilde{k})/f(\tilde{k})$ denote the equilibrium share of capital, d'Albis (2007) shows that the pair of equations (6.22a) and (6.22b) have a unique solution as long as $\lim_{\tilde{k}\to 0} s(\tilde{k}) = [0,1)$ and $\tilde{s} < \tilde{e}$, where $\tilde{e}$ is the elasticity of substitution in production and $\sigma < 1$.[6] Both conditions are mild and hold for the Cobb-Douglas production function, for example. Figure 1 illustrates this equilibrium for the calibrated model specified in Section 6.5, where AA represents (6.22a), BB depicts (6.22b), and the two intersect at the point P.

[5] Our Ξ-function is very common in the overlapping generations literature and appears in one form or the other in d'Albis (2007), Heijdra and Romp (2008), Gan and Lau (2010) and Chapters 2-4 above.

[6] These conditions have been relaxed in subsequent work by Gan and Lau (2010), who show further that uniqueness is still obtained if $\sigma \geq 1$.

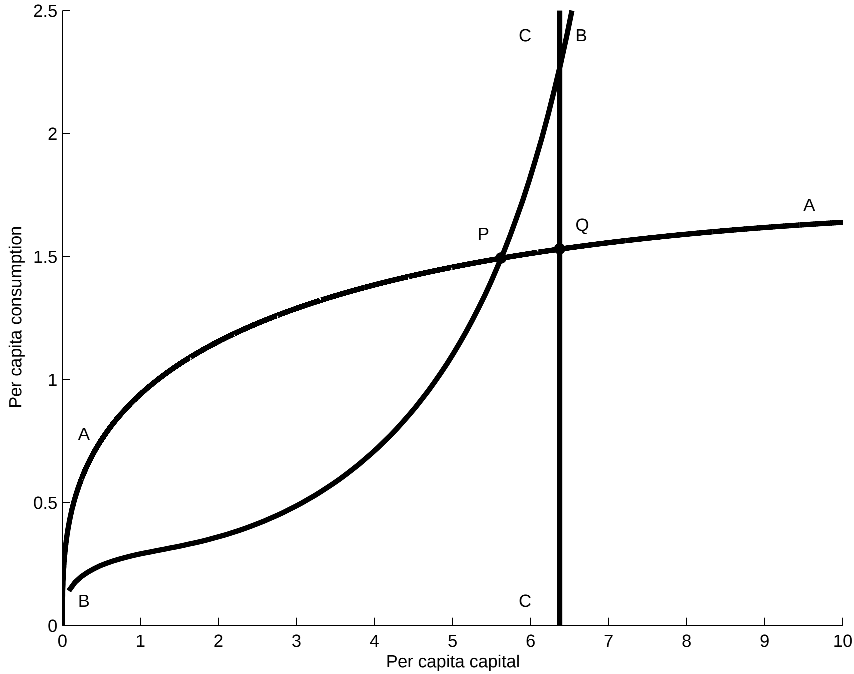


Figure 6.1. Steady-State Equilibrium

6.3.2 A 'non-demographic' steady-state

The steady-state equilibrium discussed in the previous section is the one identified by d'Albis (2007), Lau (2009), and Gan and Lau (2010). While they argue that the solution to (6.22a) and (6.22b) is unique, there is in fact a second steady-state equilibrium associated with the underlying dynamic system (6.17). This can be identified by taking the following steps: (i) substitute (6.12) into (6.12), (ii) use the result $\dot{C}(v,t) = C_v(v,t) + C_t(v,t)$, and (iii) recall (6.3), thereby enabling us to rewrite (6.12) as

$$\begin{aligned} \dot{c}(t) &= [\sigma(r(t)-\rho)-n]\left[c(t)-\beta\int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)}C(v,t)\,dv\right] \\ &\quad +\beta\int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)}\dot{C}(v,t)\,dv. \end{aligned} \tag{6.12'}$$

Now, for notational convenience, let us define

$$\begin{aligned} X(t) &\equiv c(t) - \beta \int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)} C(v,t)\, dv \text{ and, hence,} \\ \dot{X}(t) &\equiv \dot{c}(t) - \beta \int_{t-\bar{D}}^{t} e^{-n(t-v)-M(t-v)} \dot{C}(v,t)\, dv \end{aligned}$$

permitting us to express (6.23) in more compact form

$$\dot{X}(t) = [\sigma(r(t) - \rho) - n]\, X(t) \tag{6.23}$$

Recalling (6.18), equation (6.23) is seen to yield the *two* steady-state conditions

$$\text{(i) } \sigma(\tilde{r} - \rho) = n \text{ and (ii) } \tilde{c} = \beta \tilde{C}_0 \int_0^{\bar{D}} e^{(\sigma(\tilde{r}-\rho)-n)u - M(u)} du \tag{6.23'}$$

Thus, in addition to (6.22a) and (6.22b), the pair of equations

$$f\left(\tilde{k}\right) = \tilde{c} + (\delta + n)\, \tilde{k}, \tag{6.24a}$$

$$\sigma\left(f'(k) - \delta - \rho\right) - n = 0, \tag{6.24b}$$

define an alternative steady-state. This is illustrated in Fig. 1 for the calibrated model by the intersection of AA and the vertical line CC, corresponding to (6.24b), at the point, Q. The key point to observe is that this steady-state is independent of the demographic structure, except insofar as this determines the overall population growth rate through the demographic steady-state condition.

There is a sharp contrast between (6.24b) which characterizes the "non-demographic" steady-state and (6.20) [and (6.22b)], where the demographic structure plays and important role through the impact of the mortality function on the discounting of future consumption. Recalling (6.18) and (6.24b), the key feature of this 'non-demographic equilibrium' is that the steady-state growth rate of consumption across cohorts just equals the growth rate of the population. In that case, the amount of consumption given up by the dying just equals that required to sustain the consumption of the growing population.

The underlying dynamic equation (6.23) is similar in structure to equation (10) of Bommier and Lee (2003), with (6.22b) corresponding to their "balanced" equilibrium and (6.24b) corresponding to their "bubble" equilibrium.[7] Since we know that $X(t) = 0$, *for all* t, while $\sigma(r(t) - \rho) - n = 0$ holds only in the non-demographic steady-state, where $\sigma(\tilde{r} - \rho) = n$, equation (6.23) can help determine the relevance of the two steady-states. Thus, suppose the system starts out with an arbitrary aggregate capital stock, such that $\sigma(r(t) - \rho) - n \neq 0$. In this case, $X(t) = \dot{X}(t) = 0$ for all t, and eventually as the economy evolves we reach the demographic steady-state:

$$\tilde{X} \equiv \tilde{c} - \beta \int_0^{\bar{D}} e^{-nu - M(u)} C(u)\, du = 0$$

This occurs irrespective of the path of $[\sigma(r(t) - \rho) - n]$, making it clear that the demographic steady-state is in fact the relevant one.

Now consider what happens if the initial capital stock, $\tilde{k}_N$, yields $\sigma(\tilde{r} - \rho) = n$ and the economy is in the non-demographic steady-state. From (6.22a), the corresponding per capita consumption is $\tilde{c}_N$, while the implied return to capital and wage rate are respectively $\tilde{r}_N$ and $\tilde{w}_N$. Given these steady-state values, the agent's steady-state intertemporal budget constraint (6.19), and the steady-state aggregation of consumption across cohorts, (6.20) yield the following two solutions for consumption at birth, $\tilde{C}_0$:

$$\tilde{C}_0 = \tilde{w} \frac{\int_0^{\bar{D}} e^{-\tilde{r}u - M(u)} du}{\int_0^{\bar{D}} e^{-(\tilde{r} - n)u - M(u)} du} \text{ and } \tilde{C}_0 = \frac{\tilde{c}_N}{\beta \int_0^{\bar{D}} e^{-M(u)} du}. \tag{6.25}$$

However, these two solutions are, in general, inconsistent, and consequently the "non-demographic" steady-state is in general not viable.[8]

The question of viability of the bubble equilibrium is discussed by Tirole (1985) and Bommier and Lee (2003). They suggest that certain institutions, such as intergenerational transfers or money, may exist that assure imbalance in the capital market, which in their case lead to asset bubbles. In a similar vein, it may be possible to devise a suitable system of transfers that reconciles the two solutions for consumption at birth,

[7] The "bubble" steady-state was first identified by Tirole (1985) for a Diamond-Samuelson model and generalized to the continuous case by Bommier and Lee (2003, p. 146 ff.).

[8] However, the "non-demographic" steady-state does satisfy the transversality condition, so it cannot be ruled out as being unsustainable on the grounds of intertemporal insolvency.

$\tilde{C}_0$ implied by (6.25).[9]

Although we refrain from analyzing such transfers further, it is instructive to compare the two equilibria at P and Q, with the steady-state obtained in the infinitely-lived representative agent model. Denoting the corresponding steady-state per capita capital stocks by $\tilde{k}_P$, $\tilde{k}_Q$ and $\tilde{k}_R$, these three quantities are determined respectively by:

$$\sigma\left(f'\left(\tilde{k}_P\right)-\delta-\rho\right) = \frac{\tilde{\Phi}}{\tilde{c}} \tag{6.26a}$$

$$\sigma\left(f'\left(\tilde{k}_Q\right)-\delta-\rho\right) = n \tag{6.26b}$$

$$\sigma\left(f'\left(\tilde{k}_R\right)-\delta-\rho\right) = \sigma n \tag{6.26c}$$

Recalling (6.12), (6.26) implies that if (i) the total consumption given up by the dying exceeds the consumption of the newborn, and if (ii) the intertemporal elasticity of substitution, $\sigma < 1$, that $\tilde{k}_P < \tilde{k}_Q < \tilde{k}_R$.[10] Having established the basic properties of the "non-demographic" steady-state, for the remainder of the chapter we return to the analysis of the more relevant "demographic" steady-state.

6.3.3 Capital maximizing birth rate

Having established the steady-state characteristics of the model we now turn to the relationship between the aggregate capital stock and the underlying demographic structure. d'Albis (2007) argues that there exists a birth rate that maximizes the per capita capital stock. He defines the measure: $\alpha_x \equiv \int_0^{\bar{D}} up(u)x(u)du / \int_0^{\bar{D}} p(u)x(u)du$ where α_x measures the average of the quantity $x(u)$ across cohorts. He then shows that the capital stock-maximizing birth rate occurs where the average age of workers equals the average age of asset holders, i.e. $\alpha_W = \alpha_A$. In our case it is straightforward to show that:

$$sgn(\alpha_A - \alpha_W) = sgn\left(\int_0^{\bar{D}} u\tilde{A}(u)p(u)du - \int_0^{\bar{D}} up(u)du \cdot \int_0^{\bar{D}} \tilde{A}(u)p(u)du\right) \tag{6.27}$$

[9] In general, these transfer systems need to consist of an unproductive entity that transfers and collects resources from the agents in such a way that on aggregate the transfered and collected resources do not balance. The surplus or deficit of such a system may be due to capital flows to or from abroad in an open economy or, in a closed economy, an unbalanced pay-as-you-go pension system (see Bommier and Lee, 2003, p.150).

[10] In the sense that case all steady states are dynamically efficient, in that the capital stocks would be less than at the golden rule.

Using the fact that $p(u)$ may be interpreted as a probability density function we conclude that for $\alpha_W = \alpha_A$ the covariance between $\tilde{A}(u)$ and u must be zero.[11] For this to be so, assets either have to be constant over the life-cycle or their time profile has to be linearly independent of the age profile.

To see that assets are actually hump-shaped over the life cycle, rather than constant, note that in the steady state, agents accumulate assets according to:

$$\dot{\tilde{A}}(u) = (\tilde{r} + \mu(u))\, \tilde{A}(u) + \tilde{w} - \tilde{C}(u), \tag{6.28a}$$

so that starting with a zero initial capital endowment, $\tilde{A}(0) = 0$, the agent's wealth at age u is:

$$\tilde{A}(u) = \int_0^u \left[\tilde{w} - \tilde{C}(u)\right] e^{-\tilde{r}u - M(u)} du, \tag{6.28b}$$

with the transversality condition implying:

$$\int_0^{\bar{D}} \left[\tilde{w} - \tilde{C}(u)\right] e^{-\tilde{r}u - M(u)} du = 0. \tag{6.28c}$$

Under weak conditions, d'Albis shows that in this steady-state $\tilde{r} > \rho$, so that agents' consumption grows uniformly over their lifetimes.

Using this fact, in conjunction with (6.18), (6.19), and (6.28a), one can show that because $\tilde{A}(0) = \tilde{A}(\bar{D}) = 0$, $\dot{\tilde{A}}(0) > 0$, $\dot{\tilde{A}}(\bar{D}) < 0$, and that the agent's assets reach a maximum at an age $\hat{u}$:

$$\tilde{A}(\hat{u}) = \frac{\tilde{C}(\hat{u}) - \tilde{w}}{\tilde{r} + \mu(\hat{u})}.$$

Thus, the time profile of the agent's wealth over the life-cycle is hump shaped as illustrated in Panel (iii) of Figures 3-5.

As the asset profile is hump shaped over the life-cycle it may be that there exists a unique value of the birth rate such that the asset profile and the age profile are not linearly dependent. In that case the average age of asset holders equals the average age of the workers. But as savings are primarily used to finance consumption later in life, it is fair to suppose that the average age of the capital owner is higher than the average age of the worker. Indeed, in our simulations we show that for a realistic mortality

[11] The key result that is being employed is that $E(xy) = E(x)E(y) + cov(x,y)$.

function $\alpha_A = 52.65$, $\alpha_W = 43.08$. Thus, we find that an increase in the population growth rate associated with an increase in the birth rate leads to a reduction in the per capita stock of capital. In contrast, our simulations also show that if the increase in the population growth rate is the result of a reduction in mortality it will result in an increase in the per capita capita capital stock; see Table 2 and Section 5.[12] This contrast in the two ways of increasing the growth rate of population is consistent with the empirical evidence on this issue obtained by, inter alia, Kelley and Schmidt (1995), Bloom, Canning and Graham (2003) and Erlandsen and Nymoen (2008).

6.4 Specific Demographic Models

Thus far, we have not imposed any restrictions on the exact form of the survival function. To proceed further, we focus on the functional form proposed by Boucekkine, de la Croix, and Licandro (2002) labeled BCL, which was used extensively in Chapter 2-4. As we saw there, it is very tractable, amenable to numerical simulations and fits the data well (for details see Box 2.2). For comparative purposes, and to show how it fits into our analytical framework, we also discuss the familiar demographic structure proposed by Blanchard (1985), Buiter (1988) and Weil (1989) labeled BBW.[13]

6.4.1 BCL demographic structure

While the general macrodynamic equilibrium is summarized by the system (6.17), the evolution of $\Phi(t)$ may in fact be complex, requiring one to consider the dynamics of its components. To this end it is practical to begin with the alternative definition of $\Phi(t)$, given in (20b), which for the BCL function becomes:

$$\Phi(t) = \frac{\beta\eta_1}{\eta_0 - 1}\int_{t-\bar{D}}^{t} e^{(\eta_1 - n)(t-v)} \cdot C(v,t)\,dv - \beta C(t,t) + nc(t).$$

[12] In an early contribution Sinha (1986) finds the same results in a numerical simulation of the Diamond-Samuelson (DS) model.

[13] Alternatively, Bruce and Turnovsky (2010) use the de Moivre function which has the advantage of including both the DS and BBW specifications as special cases, but is less tractable than the BCL function.

Using (6.4) and (6.6) we can write:

$$\Phi(t) = \Gamma(t) - \beta \frac{H_B(t)}{\Delta_B(t)} + nc(t), \tag{6.29}$$

where:

$$\Gamma(t) = \frac{\beta\eta_1}{\eta_0 - 1} \int_{t-\bar{D}}^{t} C(v,t)\, e^{(\eta_1 - n)(t-v)} dv \tag{6.30a}$$

$$\frac{H_B(t)}{\Delta_B(t)} = C(t,t) \tag{6.30b}$$

and

$$H_B(t) \equiv H(t,t) = \int_t^{t+\bar{D}} w(\tau)\, e^{-R(t,\tau) - M(\tau - t)} d\tau, \tag{6.30c}$$

$$\Delta_B(t) \equiv \Delta(t,t) = \int_t^{t+\bar{D}} e^{(\sigma-1)R(t,\tau) - \sigma\rho(\tau-t) - M(\tau-t)} d\tau. \tag{6.30d}$$

That is, $H_B(t)$ and $\Delta_B(t)$ are, respectively, the amounts of human wealth and the inverse marginal propensity to consume *at birth.*

Differentiating (6.30a)-(6.30d), imposing the factor prices (6.16a), (6.16b), and recalling the dynamics of consumption and capital (6.17a), (6.17b), the full dynamic system can then be expressed as:[14]

$$\dot{c}(t) = \left(\sigma\left(f'(k(t)) - \delta - \rho\right) - n\right) c(t) - \frac{\beta\eta_1}{\eta_0 - 1}\Gamma(t) + \beta\frac{H_B(t)}{\Delta_B(t)} \tag{6.31a}$$

$$\dot{k}(t) = f(k(t)) - (\delta + n)\, k(t) - c(t) \tag{6.31b}$$

$$\begin{aligned}\dot{\Gamma}(t) = {} & \frac{H_B(t)}{\Delta_B(t)} - \frac{H_B(t-\bar{D})}{\Delta_B(t-\bar{D})} e^{\sigma R(t-\bar{D},t) + (\eta_1 - n - \sigma\rho)\bar{D}} \\ & + \left(\sigma\left(f'(k(t)) - \delta - \rho\right) + \mu_1 - n\right)\Gamma(t)\end{aligned} \tag{6.31c}$$

$$\begin{aligned}\dot{\Delta}_B(t) = {} & -1 - \left((\sigma - 1)\left(f'(k(t)) - \delta\right) - \sigma\rho + \mu_1\right)\Delta_B(t) \\ & + \frac{\eta_1\eta_0}{\eta_0 - 1}\int_t^{t+\bar{D}} e^{(\sigma-1)R(t,\tau) - \sigma\rho(\tau - t)} d\tau\end{aligned} \tag{6.31d}$$

$$\begin{aligned}\dot{H}_B(t) = {} & -f(k(t)) + f'(k(t))\, k(t) + \left(f'(k(t)) - \delta - \mu_1\right) H_B(t) \\ & + \frac{\eta_1\eta_0}{\eta_0 - 1}\int_t^{t+\bar{D}} w(\tau)\, e^{-R(t,\tau)} d\tau\end{aligned} \tag{6.31e}$$

[14] In determining (40d), (40e) we have used $e^{\mu_1(\tau-t)} = \mu_0 - e^{-M(\tau-t)}(\mu_0 - 1)$.

This comprises a fifth-order system in: (i) per capita consumption, (ii) per capita capital stock, (iii) the consumption given up by the dying, (iv) the initial human wealth of the new born, and (v) the (inverse) of the marginal propensity to consume out of wealth by the newborn. In principle, the dynamics can be analyzed using numerical simulations. We should note that with H_B and Δ_B being evaluated both at time t and at time $t - \bar{D}$ this involves the analysis of mixed differential-difference equations, which presents a computational challenge that is beyond the scope of the present chapter.[15] Indeed, as d'Albis and Augeraud-Véron (2009) emphasize, the characterization of the dynamics in terms of a mixed differential-difference equation is essentially generic in continuous-time overlapping generations models, one of the few exceptions being the BBW model.[16]

Steady state

Defining $\varphi\left(x, \bar{D}\right) \equiv \left(1 - e^{-x\bar{D}}\right)/x$, the steady state can be summarized by the following system[17]

A. Demographic Variables

$$\frac{1}{\beta} = \frac{1}{\eta_0 - 1}\left[\eta_0 \varphi(n, \bar{D}) - \varphi(n - \eta_1, \bar{D})\right] \tag{6.32a}$$

$$\bar{D} = \frac{\ln \eta_0}{\eta_1} \tag{6.32b}$$

B. Economic Variables

$$\tilde{C}_0 = \frac{\tilde{w}\left[\eta_0 \varphi(\tilde{r}, \bar{D}) - \varphi(\tilde{r} - \eta_1, \bar{D})\right]}{\left[\eta_0 \varphi(\sigma\rho + (1-\sigma)\tilde{r}, \bar{D}) - \varphi(\sigma\rho + (1-\sigma)\tilde{r} - \eta_1, \bar{D})\right]} \tag{6.32c}$$

$$\tilde{c} = \frac{\beta \tilde{C}_0}{\sigma(\tilde{r} - \rho) - n}\left\{\frac{\eta_1}{\eta_0 - 1}\varphi\left(\sigma(\rho - \tilde{r}) + n - \eta_1, \bar{D}\right) - 1\right\} \tag{6.32d}$$

$$f(\tilde{k}) = \tilde{c} + (\delta + n)\tilde{k} \tag{6.32e}$$

[15] In the special case of constant returns and a rectangular survival function it becomes possible to characterize the equilibrium dynamics; see, for instance, d'Albis and Augeraud-Véron (2009) and the references therein. As they point out, the representation of the equilibrium dynamics by a mixed differential-difference equation introduces oscillations into the transitional path.

[16] The reason that the BBW model can be represented by a system of ordinary differential equations is because all individuals have the same life-expectancy independent of age.

[17] We are focusing on the 'demographic equilibrium' at which $\sigma\left(\tilde{r} - \rho\right) \neq n$.

where $\tilde{r}$ and $\tilde{w}$ are defined in (6.16a) and (6.16b). Equations (6.32a) and (6.32b) define the demographic structure, summarized by the four parameters, β, η_0, η_1, and n. Given the demographic parameters and the definitions of $\tilde{r}$ and $\tilde{w}$, equations (6.32c)-(6.32e) determine the economic variables, $\tilde{C}_0$, $\tilde{c}$ and $\tilde{k}$. By combining (6.32c) and (6.32d) this can be reduced to a pair of equations in $\tilde{c}$ and $\tilde{k}$, which is analogous to (6.22a) and (6.22b). Having determined the aggregates, the steady-state age profiles of consumption and asset accumulations can be obtained by substituting (6.32c) into (6.18) and (6.28).

The system (6.32) provides the basis for our numerical simulations in Section 5. We use this system to examine the effects of a number of economic and demographic structural changes on both the aggregate behaviour of the economy and on the patterns of consumption and asset accumulation over the life cycle.

6.4.2 BBW demographic structure

For comparative purposes it is useful to show how the BBW model fits into this framework. Blanchard (1985) assumes the birth rate to be equal to the mortality rate ($\beta = \mu$), so that the net population growth rate is zero. Buiter (1988) relaxes this assumption and extends the model to the case where $\beta \neq \mu$, effectively combining the Blanchard model with that of Weil (1989).

The survival function is specified by:

$$S\left(t-v\right) \equiv e^{-M(t-v)} = e^{-\mu(t-v)}, \tag{6.33}$$

from which we immediately infer that the hazard rate, μ, is constant, while the relative cohort size is $p\left(v,t\right) = \beta e^{-\beta(v-t)}$. The demographic steady-state holds by definition, life-expectancy equals $1/\mu$ and is constant over the life cycle, while the average age of workers is $1/\beta$.

The key variable in the dynamics, the generational turnover term, $\Phi\left(t\right)$, now simplifies drastically to:

$$\begin{aligned}
\Phi\left(t\right) &= \int_{-\infty}^{t} \mu \cdot \beta e^{\beta(v-t)} \cdot C\left(v,t\right) dv - \beta C\left(t,t\right) + nc(t) \\
&= (\mu+n)c\left(t\right) - \beta C\left(t,t\right). \qquad (6.34)
\end{aligned}$$

Introducing the BBW structure into (6.6a) leads to:

$$C(v,t) = \frac{A(v,t) + \int_t^{\infty} w(\tau)\, e^{-R(t,\tau)-\mu(\tau-t)} d\tau}{\int_t^{\infty} e^{(\sigma-1)R(t,\tau)-\sigma\rho(\tau-t)-\mu(\tau-t)} d\tau} = \frac{A(v,t) + H(t)}{\Delta(t)}. \tag{6.35}$$

The crucial characteristic that renders the model so tractable is that all agents have the same planning horizon (i.e., ∞) and mortality rate (i.e., μ). Therefore, human wealth, $H(t)$, (future discounted income from labour) is the same for all agents, irrespective of their age. The same applies to $\Delta(t)$, the (inverse of) the marginal propensity to consume out of human wealth:

$$\Delta(t) = \int_t^{\infty} e^{(\sigma-1)R(t,\tau)-\sigma\rho(\tau-t)-\mu(\tau-t)} d\tau. \tag{6.36}$$

Differentiating (6.36), its dynamics are governed by:

$$\dot{\Delta}(t) = -1 - ((\sigma-1)\, r(t) - \sigma\rho - \mu)\, \Delta(t). \tag{6.37}$$

Aggregate per-capita consumption is:

$$\begin{aligned} c(t) &\equiv \int_{-\infty}^{t} p(v,t)\, C(v,t)\, dv = \int_{-\infty}^{t} p(v,t)\, [\Delta(t)]^{-1} (A(v,t) + H(t))\, dv \\ &= [\Delta(t)]^{-1} (a(t) + H(t)) = [\Delta(t)]^{-1} (k(t) + H(t)). \end{aligned} \tag{6.38}$$

From (6.6a) consumption of a new born, $C(t,t)$, is:

$$C(t,t) = [\Delta(t)]^{-1} H(t) = c(t) - [\Delta(t)]^{-1} k(t). \tag{6.39}$$

Hence, using (6.34), and recalling that $n = \beta - \mu$, we can write the aggregate dynamic system as:

$$\dot{k}(t) = f(k(t)) - (\delta + n)\, k(t) - c(t) \tag{6.40a}$$

$$\dot{c}(t) = \sigma\left(f'(k(t)) - \delta - \rho\right) c(t) - \beta[\Delta(t)]^{-1} k(t) \tag{6.40b}$$

$$\dot{\Delta}(t) = -1 - \left((\sigma-1)\left(f'(k(t)) - \delta\right) - \sigma\rho - \mu\right) \Delta(t), \tag{6.40c}$$

thus reducing it to a tractable third order system; see also Blanchard (1985, p. 234). The steady state follows readily by setting $\dot{k}(t) = \dot{c}(t) = \dot{\Delta}(t) = 0$.

6.5 Numerical Simulations

To obtain further insights, we simulate the steady-state demographic equilibrium using the BCL survival function. To do this, we first estimate its two parameters, η_0 and η_1, by nonlinear least squares, using US cohort data for 2006.[18] The estimation results reported in Table 1 highlight that we obtain a tight fit with highly significant parameter estimates. The resulting estimated survival function is illustrated in Fig. 2. Since we do not consider childhood and education, we normalize the function so that birth corresponds to age 18. As can be seen in the figure it tracks the actual survival data for the United States closely from age 18 until around 95. Beyond that age its concavity does not match the data particularly well. However, we do not view that as serious since only 1.5% of the US population exceeds 95 and these individuals are generally retired and are relatively inactive in the economy.[19] For comparative purposes we also estimate and illustrate the BBW survival function in Table 1 and Fig. 2. Being convex, rather than concave, it does does not match the data well.

Table 6.1. Estimated Survival Functions

$S(u) = I(u \leq D)\,\frac{\mu_0 - e^{\mu_1 u}}{\mu_0 - 1} + \varepsilon$ where $\varepsilon \sim i.i.d.\left(0, \sigma^2\right)^1$ $S(u) = e^{\mu u} + \varepsilon$ where $\varepsilon \sim i.i.d.\left(0, \sigma^2\right)^2$		
Demographic function	BCL[1]	BBW[2]
η_0 (st. dev.)	78.3618 (6.0193)	
$\eta_1(\mu)$ (st. dev.)	0.0566 (0.0011)	0.0112 (0.0011)
Adj. R^2	0.9961	0.6157

[1] Boucekkine, de la Croix and Licandro (2002): $I(u \leq D)$ is an indicator function that is 1 for $u \leq D$ and 0 otherwise.
[2] Blanchard (1985)-Buiter (1988)-Weil (1989)

[18] Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research, Rostock (Germany). Available at www.mortality.org or www.humanmortality.de (data downloaded on 12/10/2010).

[19] With this in mind, it might be more appropriate to refer to $\bar{D}$ as the maximum attainable economic age.

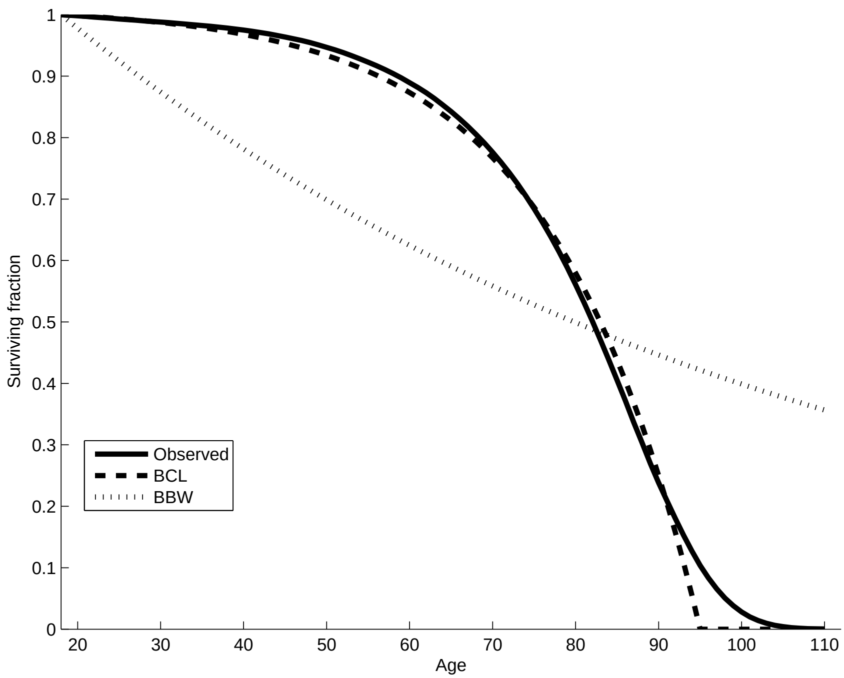


Figure 6.2. Demography

Table 2 summarizes the key structural parameters for the baseline economy, all of which are quite standard. Output is produced by a Cobb-Douglas function, $y = Ak^{\alpha}\bar{l}^{1-\alpha}$, where A is the exogenous technology index, $\bar{l}$ denotes inelastically supplied labour, with the elasticity of capital $\alpha = 0.35$ and depreciation rate $\delta = 0.05$. With respect to preferences, we set the intertemporal elasticity of substitution to 0.5, consistent with the consensus estimates reported by Guvenen (2006). We take $\rho = 0.035$ to be the rate of time preference.

The baseline calibration adopts the demographic parameters estimated above. Thus, the estimates of the BCL function imply a maximum attainable age of 95.06 and life expectancy at age 18 of 78.38. These are a little low, reflecting the fact that, as Fig. 2 illustrates, the function fails to capture the outliers beyond age 90. We take the population growth rate to be 1.00% which given the survival function, implies a birth rate of 2.24%. This is a little high because the population growth rate also takes into

Table 6.2. Baseline Parameters and Benchmark Equilibrium

Baseline Model			
Structural Parameters		BCL[1]	BBW[2]
Total factor productivity	A	1	1
Capital share of output	α	0.35	0.35
Depreciation rate	δ	5%	5%
Inter-temporal substitution elasticity	σ	0.35	0.35
Time preference rate	ρ	3.5%	3.5%
Demographic Parameters			
Youth mortality	μ_0	78.3618	N/A
Old age mortality	μ_1	0.0566	0.0112
Birth rate (implied)	β	2.24%	2.12%
Life-expectation at 18 (Age)	L_{18}	78.38	89.29
Average age of workers	α_W	43.08	32.76
Average age of asset holders	α_A	52.65	61.29
Maximum attainable age (implied)	$\bar{D}$	95.06	∞
Population growth rate	n	1.00%	1.00%
Implied Economic Variables			
Per capita capital stock	$\tilde{k}$	5.6226	7.4044
Per capita output	$\tilde{y}$	1.8301	2.0152
Capital/ Output ratio	$\tilde{k}/\tilde{y}$	3.0722	3.6742
Real interest rate	$\tilde{r}$	6.39%	4.52%
Wage rate	$\tilde{w}$	1.1896	1.3099
Average per capita consumption	$\tilde{c}$	1.4928	1.5710
Marginal propensity to consume at birth	$[\Delta_B]^{-1}$	0.053	0.051

[1]Boucekkine, de la Croix and Licandro (2002)

[2]Blanchard (1985)-Buiter (1988)-Weil (1989)

account immigration. The implied equilibrium economic variables include an equilibrium capital-output ratio of 3.07 and a real net return on capital of 6.39%. The marginal propensity to consume at birth out of wealth is approximately 0.053%, and the each cohort's consumption grows at 1.45% with age. The corresponding parameters and implied equilibrium values for the BBW model are also reported in Table 2. It yields a much higher life expectancy, due to the fact that the maximum attainable age in that model is infinite.

From this initial baseline equilibrium we analyze the steady-state effects of two types of structural changes: (i) an increase in productivity; (ii) changes in the demographic structure.

6.5.1 Increase in productivity

We consider a neutral technological change, where A increases by 25% from 1 to 1.25. As seen from Row 2 in Table 3, this leads to a proportionate increase in capital and output, causing the capital-output ratio to remain unchanged.

Fig. 3.A illustrates the aggregate and the distributional effects for the BCL survival function. The locus BB in panel (i) depicts the pre-shock growth in consumption with age (eq. (6.3)). The increase in productivity raises the wage rate, while the rate of return on capital remains unchanged. This causes the BB locus to shift up to B′B′, implying a uniformly higher consumption level for all ages, but growing at the unchanged rate. The AA locus presents the average per capita consumption, which correspondingly jumps up to A′A′. Panel (ii) illustrates the long-run distributional changes across the cohorts. Its mildly hump-shaped locus reflects the fact that the increase in consumption with age is offset by the increasing mortality with age, leading to declining cohort-weighted consumption.

Panel (iii) illustrates the distribution of assets along the life cycle. Starting with zero assets at birth (18), agents accumulate wealth until around 70, after which they decumulate until assets run out at the maximum attainable age. This is reflected in the inverted-U locus EE which shifts out to E′E′ with the increase in productivity. The figures indicate that the greatest impact on wealth of the productivity increase accrues to individuals aged around 70. The upward shift in the distributional locus is also reflected in the horizontal line DD which illustrates the average per capita wealth, and which shifts up to D′D′ following the technological increase. Panel (iv) reflects assets weighted by the size of the cohorts. Due to the decline in survival with age the greatest share of the benefits is enjoyed by the 55 year old cohort.

Fig. 3.B illustrates the same exercise for the BBW demographic structure. It contrasts sharply, and is much less plausible, as a result of the convex survival function and the fact that agents may potentially live indefinitely (albeit with an arbitrarily low probability). For example, the perpetual upward slope of the assets accumulation locus EE in panel (iii) is unsatisfactory. However, with the dwindling cohort size the implications for distributions across cohorts, as illustrated in Panel (iv) is closer to the pattern implied by the more plausible BCL survival function.

Table 6.3. Structural Changes

		Demography		Economic Variables						
		L_{18}	n	$\tilde{k}$	$\tilde{y}$	$\tilde{k}/\tilde{y}$	$\tilde{r}$	$\tilde{w}$	$\tilde{c}$	$[\Delta_B]^{-1}$
Baseline Model		78.38	1.00%	5.623	1.830	3.072	6.39%	1.190	1.493	0.053
Increase in productivity	$A \to 1.25$	78.38	1.00%	7.926	2.580	3.072	6.39%	1.677	2.104	0.053
Demographic Shocks										
Increase in the birth rate	$\beta \to 2.57\%$	78.38	1.50%	5.540	1.821	3.042	6.50%	1.184	1.461	0.054
Decrease in youth mortality	$\eta_0 \to 195$	93.98	1.50%	6.125	1.886	3.248	5.78%	1.226	1.488	0.049
Decrease in old age mortality	$\eta_1 \to 0.0443$	95.15	1.50%	6.211	1.895	3.278	5.68%	1.232	1.491	0.048
Off-setting change in birth rate and old age mortality rate	$\beta \to 2.53\%$ $\eta_1 \to 0.00551$	80.03	1.50%	5.623	1.830	3.072	6.39%	1.190	1.465	0.053

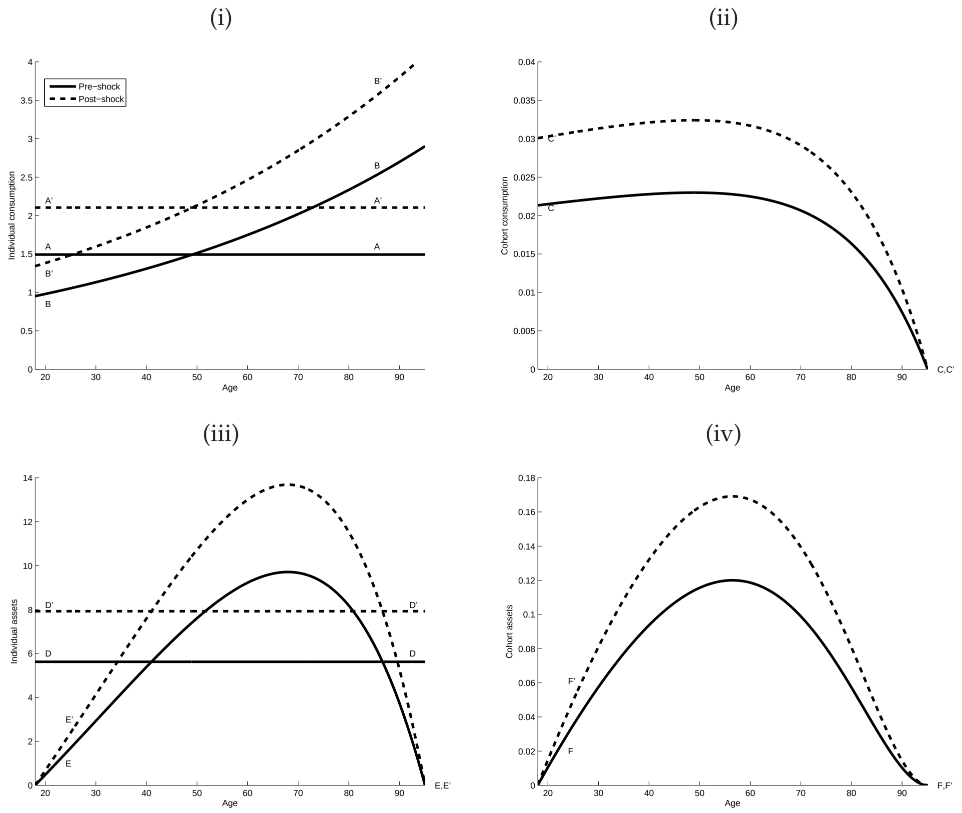


Figure 6.3. Increase in Productivity: BCL

6.5.2 Changes in the demographic structure

We contrast the impact of an increase in the population growth rate of 0.5 percentage points driven by either an increase in the birth rate, a decrease in mortality, or a combination of the two. Table 3 summarizes the various scenarios and shows how the economic consequences differ dramatically, depending on the source of the increase in the population growth rate.

Increase in the birth rate

In order to increase the population growth rate by 0.5 percentage point from 1.00% to 1.50% the birth rate must increase from 2.24% to 2.57%. Table 3, line 4 reveals that

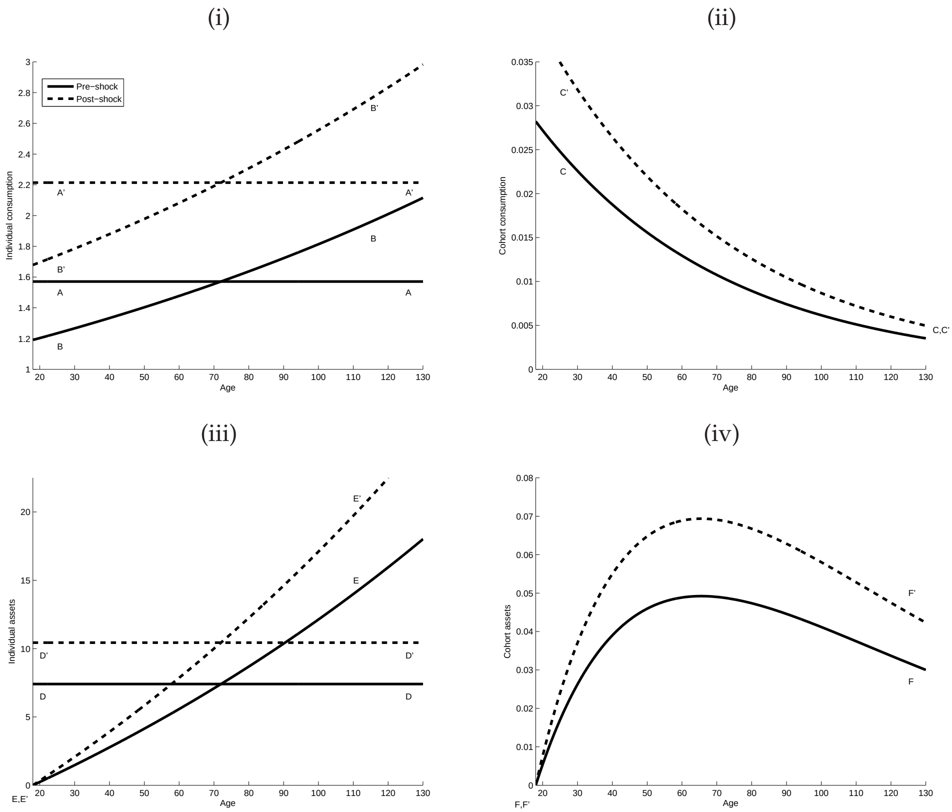


Figure 6.4. Increase in Productivity: BBW

this leads to a 1.48% reduction in the per capita capital stock (from 5.623 to 5.540). This is illustrated by the slight downward shift of the line DD in Fig. 5, Panel (iii). This response is consistent with the characterization of the steady state provided in Section 3 and the fact that the average age of wealth owners (52.65) exceeds that of workers (43.08). It is also consistent with the view emphasized by Kelley and Schmidt (1995) that an increase in the population growth rate resulting from a higher birth rate will have a negative effect on the level of economic activity. This is because it increases the relative number of young who have not accumulated any capital stock to contribute to the productive capacity of the economy. This reduction in aggregate assets accumulation has several consequences. It leads to a 0.5% reduction in the wage rate (from 1.190 to 1.184) and an increase in the rate of return on capital from 6.39% to

6.50%. It also leads to a 0.5% decline in per-capita output (from 1.830 to 1.821) and a 2.1% decline in per capita consumption (from 1.493 to 1.461), the latter being illustrated by the downward shift in the AA line in Fig. 5(i).

The distributional consequences are also modest, as Fig. 5 illustrates. The life cycle path for consumption, illustrated by BB in Panel (i), remains virtually unchanged. The slight reduction in the wage, with the anticipation of the future higher return to capital causes a very slight reduction in consumption at birth. However, the increase in the rate of return on capital increases the consumption growth rate over the life-cycle. Hence, toward the end of their life-cycle agents experience an increase in their consumption while average per capita consumption declines. The distributional consequences across cohorts are more substantial and in fact opposite to those experienced by individuals, as illustrated by the rotation of the CC curve to CC′ in Panel (ii). Thus, the increase in the relative size of the younger cohorts, due to the higher birth rate, implies that they enjoy a larger share of the overall consumption, while the decline in the relative size of older cohorts means that their share of consumption declines, even though each surviving member′s consumption level has increased.

The hump-shaped locus EE in Panel (iii), which reflects that the accumulation of assets over the life cycle shifts out, albeit slightly. This is a consequence of the increased rate of return on capital. Panel (iv) illustrates how, with the increase in the relative size of the young cohorts due to the higher birth rate, the share of wealth each existing cohort owns increases. This also explains why, even though at each age each individual has a slightly higher level of wealth, per capita wealth is nevertheless smaller. This is because with a higher birth rate a relatively larger share of the agents is young and as young agents posses relatively little capital, this leads to lower aggregate per-capita capital (see Panel (iii) locus DD and D′D′).

6.5.3 Decrease in the mortality rate

The two alternative ways to increase the population growth rate from 1% to 1.5% are either to decrease youth mortality, η_0, to 195 or old age mortality, η_1, to 0.0443. As the economic consequences are similar, we restrict attention to the latter.

From Table 3 we see that this leads to a 10.46% increase in the per capita stock of

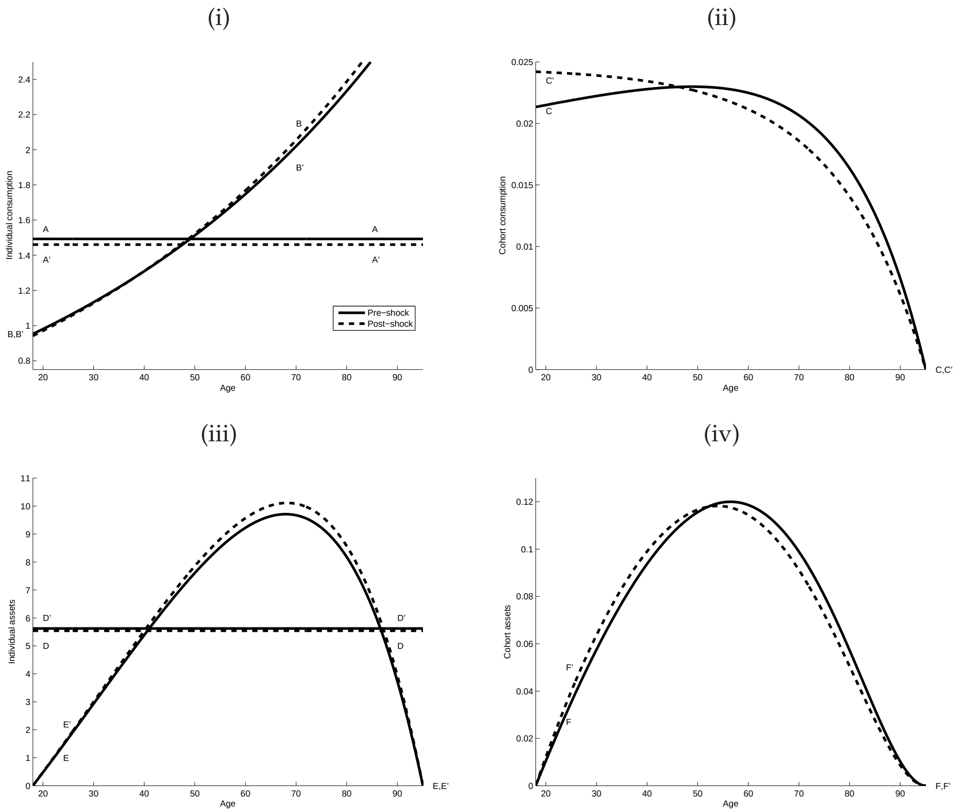


Figure 6.5. Increase in Birth Rate

capital (from 5.623 to 6.211). This is illustrated by the upward shift of the line DD in Fig. 6, Panel (iii). This response is consistent with the view emphasized by Kelley and Schmidt (1995) that an increase in the population growth rate resulting from a reduction in mortality will have a positive effect on the level of economic activity. This is because it increases the relative number of old people who have accumulated capital stock to contribute to the productive capacity of the economy. This increase in aggregate asset accumulation has several consequences. It leads to a 3.5% increase in the wage rate (from 1.190 to 1.232) and a decrease in the rate of return on capital from 6.39% to 5.68%. It also leads to a 3.5% increase in per-capita output (from 1.830 to 1.895) and a negligible (0.02%) decline in per capita consumption with the increased population, the latter being illustrated by the imperceptible shift in the AA line in Fig.

6(i).

The distributional consequences are illustrated in Fig. 6 and are seen to be non-monotonic. Panel (i) shows that the increase in the wage rate coupled with the anticipation of the future lower return to capital causes a slight increase in consumption at birth. However, the decrease in the rate of return on capital decreases the consumption growth rate over the life cycle. Hence, after a few years agents experience a decrease in their consumption and since this is the experience of most cohorts, average per capita consumption declines. In Panel (ii) we see that the increase in longevity and associated increase in old age cohorts, coupled with the upward shift and flattening of the BB curve, causes the CC curve to move out to C′C′. Thus, the increase in consumption of the very young causes their share of overall consumption to increase. However, the decline in the growth rate of consumption for people between around 30 and 80 causes their share of consumption to decline, while the increase in longevity leads to an increase in consumption share of the very old.

Panel (iii) reveals that the increase in longevity causes the EE locus to shift up and to the right. In early stages the life cycle the rate of asset accumulation declines very slightly, reflecting the decline in the rate of return on capital. As a result, the decline in mortality causes relatively young agents' wealth to decline slightly. However, the increase in longevity induces them to save for a longer period and to accumulate more assets in light of their increased longevity. Finally, Panel (iv) illustrates how the increase in the relative size of old cohorts tilts the share of wealth significantly in their direction.

These patterns are consistent with the empirical evidence. For example, the fact that consumption declines for all but the youngest cohorts, while the wealth of older agents increase is consistent with the empirical findings of Fair and Dominguez (1991), Attfield and Cannon (2003), and Erlandsen and Nymoen (2008) all of whom find that the effect of an aging population is to lead to a decline in overall per capita consumption for all equivalent income levels. The pattern we obtain of asset accumulation increasing with life expectancy agrees with the findings of Bloom, Canning, and Graham (2003).

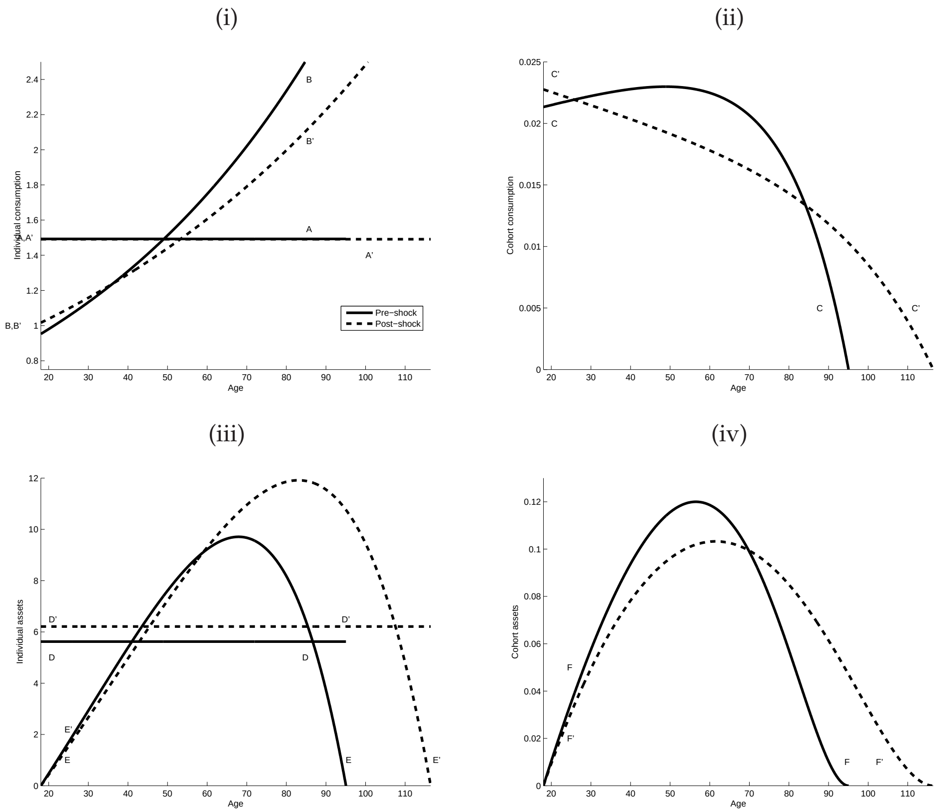


Figure 6.6. Decrease in Old Age Mortality

6.5.4 Increase in birth rate versus decrease in mortality

Comparing Figs. 4 and 5 we see that achieving a specified increase in the population growth rate by increasing the birth rate or decreasing the mortality rate has dramatically different consequences for the economy. First, whereas only a mild increase in the birth rate of 0.33% will raise the population growth rate by 0.5%, to achieve the same objective by reducing mortality would require increasing longevity by around 17 years, which would seem to be a much more formidable task. Second, whereas a 0.5 percentage point increase in the population growth rate resulting from an increase in births will have only a slight negative effect on the productive capacity of the economy (measured by its per capita capital stock), the same increase in the population growth rate brought about by reduced mortality will have a significant expansionary effect. This contrast in magnitudes agrees exactly with the empirical results obtained by Blanchet (1988), thus emphasizing the importance of the form in which population growth occurs.

Finally, our results can be reconciled with the cross-country empirical evidence cited by Kelley and Schmidt (1995) who found that, whereas population growth had had a negligible effect on growth during the 1960s and 1970s, it had a negative effect in the 1980s. This can be explained by comparing line 7 of Table 3 with line 4. Increasing the birth rate to only 2.53% and reducing old age mortality to 0.0551 causes the economic effects to be largely offsetting so that the per capita capital stock, output, wage rate, return to capital all remain unchanged. In summary, the changing mix between increased birth rate and decreased mortality can very naturally account for the different empirically estimated long-run effects of population growth rates at different stages of development.

6.6 Conclusions

This chapter has introduced a realistic age-dependent demographic structure into a neoclassical growth model for a closed economy. In doing so, we have had two primary objectives. The first is to provide a general characterization of how the demographic structure impedes on the macrodynamic equilibrium. We show how this depends on the generational turnover term, which is an integral component of the intertemporal consumption allocation decision. Setting up the aggregate dynamics as a generalization of the conventional neoclassical growth model, provides two major insights. Not only does it enable us to view alternative demographic specifications in a unified way, but also we are able to identify two, rather than just one, steady-state equilibria. The first is highly sensitive to the demographic structure, whereas in the second equilibrium demographic factors play but a minor role. However, in the absence of intergenerational transfers, the latter is not relevant, and therefore has not been considered further.

The second objective is to analyze the effect of structural changes – most important demographic structural changes – on both the aggregate macro equilibrium, as well as the distributional life-cycle implications. This is done numerically using the very general survival function proposed by Boucekkine, de la Croix, and Licandro (2002). The most striking result is the sharp contrast, both qualitatively and quantitatively, in the effects of changes in the population growth rate on the macro economy. Whether an increase in population occurs because of an increase in births or a decrease in mortality is crucially important, and in this regard our results corroborate the empirical findings obtained in the demographic literature.

While this chapter is mainly theoretical and quantitative, it clearly can be extended in various directions. First, the contrast between births and mortality in influencing the population growth rate and the resulting consequences for distribution across cohorts and for the aggregate economy raises interesting policy issues for a country seeking to influence its population growth rate. Second, it is straightforward to extend the framework to allow for retirement and to address issues pertaining to social security and retirement benefits, issues that are of crucial importance for the US and other countries with their ageing populations. Finally, while we have focused on the long-run (steady-state) implications of demographic structural changes, the nature of the transition from one steady-state to another is also important.

Chapter 7

Conclusion

In this thesis we have developed a series of small macroeconomic models to analyse a variety of issues relating to the macroeconomics of ageing. In developing these models we have tried to balance the necessity of having solid microfoundations against the ability of clearly seeing what the driving forces are in the interaction between individual decisions and aggregate outcomes. In this part we will briefly summarize the main conclusions, discuss some of the limitations of the used models and touch upon areas for future research.

In the second chapter we developed a continuous-time overlapping generations model featuring single-sector endogenous growth in the spirit of Romer (1989). Using this model we took the Blanchard (1985)-Yaari (1965) assumption of the existence of a perfectly competitive annuity market by the horns. Annuities are life-insured financial products that pay out conditional on the survival of the individual agent. The annuity firm pays a premium to the annuity holder, which, if the annuity is priced actuarially fair, is equal to the individual's instantaneous probability of death. In return, the annuity firm receives the individual's assets upon his/ her death. We introduced an imperfectly competitive annuity market into the model by allowing the return received on annuities to be less than actuarially fair. The imperfection leads individual agents to discount their future utility by their instantaneous probability of death. This in turn lowers the incentive to save because agents anticipate that they might not live to benefit from their savings. If all agents save less, the growth rate of the economy decreases because capital accumulation is the driving force behind economic development in Romer's (1989) growth model. For the annuity firms the imperfectly priced annuities mean that they are making pure profits, which we let the government tax away and then redistribute equally over all agents. In terms of magnitude, we found that the impact of annuity market imperfections on economic growth is mild if proper account is taken of both age-dependent mortality and productivity.

In the third chapter we used the model to analyse the impact of, and difference between, consumption and labour-income taxes. In addition to introducing the taxes, we extended the model by allowing the redistribution of tax income (which also includes the profits of the annuity firms) to be age-dependent. We found that consumption taxes redistribute assets from the elderly, who are strong consumers, to the young, who barely consume but save a lot. At the aggregate level the redistribution of assets between non-savers and savers leads to more growth because aggregate capital accumulation is spurred. The labour-income tax, on the other hand, redistributes assets from working to retired individuals. Thus, the labour-income tax induces a redistribution of assets between saving workers and non-saving retirees. Needless to say, the growth impact of this redistribution is negative. Finally, through the same mechanism, a regime in which taxes are redistributed with a bias toward young agents leads to a higher growth rate than a regime in which taxes are redistributed to the elderly.

In the fourth chapter we used the model developed in the first chapter once more in order to analyse the moderating role of public pensions during a demographic shock that decreases the mortality rate. We simplified the model by making the labour supply decision exogenous but extended the model to allow for a public pension system. The public pension system can be run on either a defined benefit or a defined contribution basis. In addition, the government can use the retirement age as a policy variable. We found that, in general, a decrease in the mortality rate increases the economic growth rate because individuals need to accumulate more assets for their retirement period. However, if the public pension is run on a defined benefit basis the growth effect is smaller because the contribution rate for the pension has to adjust to accommodate the larger amount of pensions that have to be paid out. Surprisingly, we also found that an increase in the retirement age decreases the economic growth rate of the economy. This is a direct consequence of the fact that a higher retirement age implies a shorter retirement period and, hence, less assets necessary to make it through retirement.

The analysis of chapters 2-4 showed that substantial mileage can be achieved by using small macroeconomic models. Indeed, we saw that both theoretical issues and questions of interest to public policy can be analysed meaningfully. The proverbial tank is, however, not empty yet. In its current form the model can easily be extended to allow for an initial period in which individuals engage in human capital formation. Such an extension would shed more light on the role of individual productivity over the life-cycle. Naturally, in such a model there is an important role for financial frictions once more. After all, if individuals are born bare of assets and no loans are available from the banking sector, how should they finance their human capital investment? A more formidable, yet equally important, task is to study the transitional dynamics underlying the various policy changes which we considered. Currently we focus on a comparison between steady states while the transition between them need not be monotonic. Although we touched up the issue of transitional dynamics in later chapters, their lion's share remain open.

While in chapters 2-4 we focused on the role of annuity market imperfections for individual and aggregate outcomes, in the fourth chapter we turned to the question of whether annuities increase aggregate as well as individual welfare. To facilitate this analysis we stepped away from the continuous time overlapping generations model used in chapters 2-4 and used a smaller, more stylized model. The model builds on the canonical Diamond (1965)-Samuelson (1958) model in which agents live for two periods. One period of working (and saving) and one period of retirement (and running down savings). In order for there to be a meaningful role for annuity markets we let the transition between the two periods of life be uncertain. That is, individuals face a positive probability of death at the end of the first period. In the absence of annuities, individuals that die after the first period of life leave an accidental bequest. On the firm side of the model we allowed for a more general production structure that allows for both endogenous and exogenous growth.

In the initial analysis we assumed that the government taxes these bequests away. Within this set-up we compared and contrasted various ways in which the government can use its tax income. In order not to muddle the analysis we focused on three distinct regimes: either the government distributes the funds to the young or to the elderly, or the government uses them for unproductive spending. In this regard we found that the regime in which the government redistributes the assets toward the young outperforms the other two systems in both welfare and growth terms. The mechanism driving these effects is that the young save a lion's share of their additional funds so that redistribution towards them increases the capital stock. Somewhat surprisingly, the regime where the government wastes the bequests outperforms the system in which the bequests are transferred to the elderly. Intuitively, if the bequests are transferred to the elderly, the young save less because they anticipate a large transfer in old age.

Starting from any of the three redistribution scenarios we studied the impact of opening a perfectly competitive annuity market. That is, a market in which annuities are priced actuarially fair. From an individual perspective the annuity market offers a financial product with a superior return. Therefore, it is perfectly rational for the individuals to annuitize their assets. From an aggregate perspective, however, matters are, however, dramatically different depending on how the accidental bequests were initially redistributed. If the bequests were initially redistributed toward the young we find what we call "the tragedy of annuitization". That is, although full annuitization of assets is individually optimal it is not socially beneficial, in terms of growth and welfare, due to adverse general equilibrium repercussions. By opening up the annuity market, the young lose the wealth transfer, part of which they save for their retirement. In addition, the higher return received on the savings reduces their incentive to save. In concert these two effects assure that all generations but the initial one are worse off. In a similar vein to the results on the redistribution, we find that the tragedy also arises if the bequests were initially wasted by the government. In this respect it is unsurprising that the tragedy does not arise if the bequests were initially redistributed to the elderly.

The analysis of annuity markets in this chapter highlighted once again the insights that can be gained from employing small macroeconomic models. By eliminating all but the essential features of the individual life-cycle we were able to not only compare steady-states but also the transitional dynamics between the regimes. Starting from the bare model a very natural next step would be to allow the overlapping generations to be interconnected through the introduction of intentional bequests. In such a set-up there would be substantial intergenerational transfers even if annuity markets are present. A second interesting extension would be to combine the current model with that of the earlier chapters. Hereby we should be able to get a better feel for the magnitudes involved in the growth and welfare consequences of opening op an annuity market.

In the final chapter, we returned to the analysis of demographic shocks by focusing on different sources of demographic change as a driving force behind the economic consequences of an increase in the population growth rate. To analyse these issues we developed a continuous-time overlapping generations model akin to the model used in chapters 2-4. Although the model used in this chapter was similar to the previously used model, it differed in a number of key ways. Most important, we stepped away from Romer (1989) endogenous growth model and no longer allowed the annuity market to be imperfect. In addition, we simplified the choices faced by the individual household by letting labour supply and productivity be constant over the life-cycle. Naturally, we did keep the realistic mortality structure intact. These changes to the model allowed us to provide a deeper analysis of the equilibrium structure and dynamics governing the model. In this regard, we found that there is in fact a second equilibrium that is often overlooked in the literature. This equilibrium is, however, not sustainable in the absence of international transfers in an open economy or an unbalanced pension system in a closed economy. Thus, we focused on the more common equilibrium in the remainder of the analysis.

In the numerical analysis we showed that, depending on the source of demographic change, an increase in the population growth rate can increase, decrease or not affect the aggregate per-capita capital stock at all. If the population growth rate increases due to an increase in the birth rate, the capital stock present in the economy must be divided over more agents, which leads to a dilution of the per-capita capital stock. On the other hand, if the increase in the population growth rate is due to a decrease in the mortality rate, the agents will save more, thus increasing the per-capita capital stock. Finally, if the increase in the population growth rate is due to a combination of an increase in the birth rate and a decrease in the mortality rate, it is possible that the two effects exactly offset each other.

In close relation to the other chapters, the final chapter emphasized once more the usefulness of small macroeconomic models in the analysis of demographic change. In its current form the analysis has been able to shed light on interesting issues regarding the theoretical and quantitative structure of the model. The natural next step is to enrich the model with a meaningful government sector and analyse questions relating to public policy. A more substantial, and equally interesting, extension is to make fertility decisions or the mortality rate depend on the choices made by the individuals. In this way, we can shed more light on the interplay between the economic and demographic structure of an economy. Naturally, such an extension opens up alleys for public policy research that we have neither been able to address yet in this chapter nor the others.

This completes this section and, thereby, this thesis. The aim of this thesis was clear from the start: to contribute to the challenge of developing macroeconomic models that, on the one hand, are solidly founded in the microeconomic environment of the individual agent and, on the other hand, are able to show the analyst which main mechanisms are at play. It is up to the reader to decide whether a contribution to this challenge has been delivered.

Chapter 8

Samenvatting (Dutch Summary)*

* I thank Jacques Don, Ryanne van Dalen and Robert de Vries for help with the Dutch summary. Without them it would have been English with Dutch words.

In deze dissertatie ontwikkelen we verscheidene kleine macro-economische modellen waarmee we een aantal thema's bestuderen, gericht op de economische gevolgen van de vergrijzing. Bij het ontwikkelen van deze modellen hebben we telkens de noodzaak van solide micro-funderingen afgezet tegen de mogelijkheid om een duidelijk zicht te krijgen op de drijvende krachten bij de interactie tussen individuele besluiten en geaggregeerde uitkomsten.

Na het inleidende hoofdstuk hebben we in het tweede hoofdstuk een continue-tijd-overlappende-generatiesmodel ontwikkeld met endogene groei in de zin van Romer (1989). Met dit model hebben we de Blanchard (1985)-Yaari (1965) aanname van het bestaan van perfect competitieve markten voor annuïteiten bij de horens gevat. Deze annuïteiten zijn financiële producten die uitbetalen conditioneel op het overleven van de individuele agent. De levensverzekeraar betaalt een premie die, als de annuïteit actuarieel eerlijk geprijsd is, gelijk is aan de sterftekans van de levensverzekeringsnemer. In ruil daarvoor krijgt de levensverzekeraar de bezittingen van de levensverzekeringsnemer als deze sterft.

Wij modelleren annuïteitenmarktimperfecties door aan te nemen dat de premie die ontvangen wordt voor de annuïteiten niet actuarieel eerlijk is. Deze imperfectie leidt ertoe dat agenten hun toekomstig nut verdisconteren met hun sterftekans. Dit zorgt ervoor dat de prikkel om te sparen afneemt, omdat agenten erop anticiperen dat ze wellicht de dag niet zullen zien waarop zij van hun gespaarde vermogen kunnen genieten. Aangezien alle agenten minder sparen, zal de groeivoet van de economie dalen omdat kapitaalaccumulatie de drijvende kracht is achter economische ontwikkelingen in het model van Romer (1989). Voor de levensverzekeraars betekenen de imperfect geprijsde annuïteiten dat zij winst maken. Wij nemen aan dat deze winst door de overheid wordt wegbelast en herverdeeld over alle agenten. In termen van omvang concluderen wij dat de economische gevolgen van imperfect geprijsde annuïteiten klein zijn, als voldoende rekening gehouden wordt met zowel leeftijdsafhankelijke overlevingskansen als arbeidsproductiviteit.

In het derde hoofdstuk gebruiken we het model om de gevolgen van en verschillen in consumptie- en loonbelastingen te analyseren. Behalve met de introductie van belastingen breiden wij het model uit door de herverdeling van belastingen (inclusief de winsten van de levensverzekeraars) leeftijdsafhankelijk te maken. In dit hoofdstuk concluderen wij dat consumptiebelastingen vermogen herverdelen van ouderen die veel consumeren, naar jongeren die weinig consumeren maar veel sparen. Op geaggregeerd niveau leidt deze herverdeling van niet-spaarders naar spaarders tot meer groei omdat de kapitaalaccumulatie wordt bespoedigd. De loonbelasting daarentegen herverdeelt vermogen van werkende naar niet-werkende individuen. Als gevolg daarvan herverdeelt de loonbelasting vermogen van spaarders naar niet-spaarders. Uiteraard zijn de gevolgen van deze herverdeling negatief voor de economische groei. Onze laatste bevinding is dat, door hetzelfde mechanisme als voorheen, een scenario waarin belastingen relatief meer naar jongeren herverdeeld worden, beter is in termen van economische groei dan een scenario waarin belastingen relatief meer naar ouderen worden herverdeeld.

In het vierde hoofdstuk gebruiken wij het model uit het tweede hoofdstuk om te analyseren hoe het sociale zekerheidsstelsel de gevolgen van demografische veranderingen beïnvloedt. We breiden het model uit door een sociale-zekerheidsstelsel op te nemen maar simplificeren het model enigszins door de beslissing over het arbeidsaanbod exogeen te maken. Het sociale- zekerheidsstelsel kan of op basis van beschikbare uitkeringen of op basis van een systeem van beschikbare premies uitgevoerd worden. Daarnaast kan de overheid de pensioengerechtigde leeftijd als beleidsinstrument gebruiken. Onze bevindingen zijn dat een afname van de sterftekans leidt tot een toename van de economische groei omdat individuen meer vermogen moeten accumuleren voor hun pensioen. Echter als het sociale-zekerheidsstelsel wordt uitgevoerd op basis van beschikbare uitkeringen, is het groei-effect kleiner omdat de premies voor de pensioenen moeten worden aangepast vanwege het grotere aantal pensioenen dat nu uitbetaald moet worden. Tot onze eigen verbazing concluderen wij dat een verhoging van de pensioengerechtigde leeftijd leidt tot een daling van de economische groei. Dit is een direct gevolg van het feit dat een hogere pensioengerechtigde leeftijd een kortere pensioenperiode impliceert waardoor minder vermogen nodig is voor het pensioen.

Terwijl wij ons in hoofdstukken 2-4 hebben beziggehouden met imperfecties op de annuïteitenmarkt gaan we in het vijfde hoofdstuk in op de vraag of annuïteiten zowel de individuele als de algemene welvaart verhogen. Om deze analyse te faciliteren stappen wij af van het continue-tijd-overlappende-generatiesmodel en gebruiken een kleiner, meer gestileerd, model. Het model bouwt voort op het twee-periodenmodel van Diamond (1965) en Samuelson (1958). Dit model kent een periode voor werken (en sparen) en een periode voor pensioen (en het afbouwen van besparingen). Aan de bedrijvenkant van het model veronderstellen wij een algemenere productiestructuur die zowel endogene als exogene groei bevat. De annuïteitenmarkt krijgt een betekenisvolle rol doordat de transitie tussen de beide perioden onzeker is. Dit houdt in dat individuen worden geconfronteerd met een positieve overlijdenskans na de eerste periode. Indien individuen overlijden, en er geen annuïteiten aanwezig zijn, wordt ongewild een erfenis achtergelaten.

In de initiële analyse nemen wij aan dat erfenissen worden wegbelast door de overheid. Binnen dit raamwerk vergelijken wij verschillende manieren waarop de overheid haar belastinginkomsten kan gebruiken. Om de analyse overzichtelijk te houden richten wij ons op drie duidelijk verschillende regimes: de overheid geeft haar inkomsten geheel aan de jongeren, geheel aan de ouderen of gebruikt ze volledig voor improductieve bestedingen. Wij concluderen dat het regime waarbij de overheid haar inkomsten aan de jongeren geeft, de andere twee regimes overtreft in zowel groei- als welvaartstermen. Het mechanisme achter deze effecten is dat de jongeren een groot deel van hun additionele fondsen sparen. Met enige verbazing constateren wij dat het regime waarin de overheid haar inkomsten improductief besteedt, beter is dan het regime waarin de overheid haar inkomsten aan de ouderen geeft. De intuïtie hierachter is dat jongeren minder sparen als zij in de toekomst een overdracht verwachten en dat daardoor de kapitaalgoederenvoorraad afneemt als de overheid haar inkomsten aan de ouderen geeft.

Met de drie herverdelingsregimes als uitgangspunt bestuderen wij de gevolgen van het openen van een perfect competitieve annuïteitenmarkt. Ten opzichte van reguliere besparingen biedt de annuïteitenmarkt een superieur financieel product en daarom is het vanuit welvaartsoogpunt voor individuen rationeel om al hun bezittingen te annuitiseren. De zaken zijn echter heel anders vanuit een algemeen perspectief afhankelijk van hoe in de uitgangssituatie de erfenissen werden herverdeeld. Als de erfenissen eerst aan de jongeren werden gegeven, constateren wij wat wij de ”tragedie van annuitiseren” noemen. Dat wil zeggen, terwijl annuitiseren individueel optimaal is, is het in sociaal opzicht niet optimaal vanwege negatieve algemene evenwichtseffecten. Door het openen van de annuïteitenmarkt verliezen de jongeren de overdracht waarvan ze een groot deel spaarden voor hun pensioen. Bovendien vermindert de hogere rente op annuïteiten de prikkel om te sparen. Deze twee effecten zorgen er samen voor dat, op de eerste generatie na, alle generaties op een lager welvaartsniveau zitten. Het is met het oog op de resultaten over de verschillende herverdelingsregimes niet opmerkelijk dat de tragedie eveneens optreedt als de erfenissen eerst voor improductieve bestedingen werden gebruikt. Met dezelfde redenering kan eveneens worden geconcludeerd dat de tragedie niet optreedt als de erfenissen eerst naar de ouderen gingen.

In het laatste hoofdstuk keren we terug naar de analyse van demografische schokken door te kijken naar de wijze waarop verschillende oorzaken van demografische veranderingen de economische gevolgen van een verandering in de bevolkingsgroei beïnvloeden. Om deze zaken te analyseren hebben we een continue-tijd-overlappende-generaties model ontwikkeld dat lijkt op het model dat we in de hoofdstukken 2-4 hebben gebruikt. Terwijl dit model lijkt op het eerder gebruikte model, wijkt het er op een aantal belangrijke punten van af. Als belangrijkste verschil zijn we afgestapt van het endogene groeimodel van Romer (1989) en laten we niet langer toe dat de annuïteitenmarkt imperfect kan zijn. Bovendien hebben we de keuzes van de huishoudens gesimplificeerd door arbeidsaanbod en productiviteit van de huishoudens constant te houden gedurende de levensloop. Deze veranderingen zorgen ervoor dat we het evenwicht en de dynamiek van het model beter kunnen analyseren. Met betrekking hiertoe hebben we gevonden dat er een tweede evenwicht bestaat dat veelal over het hoofd wordt gezien in de literatuur. Echter, dit evenwicht is niet houdbaar als er geen internationale overdrachten of een ongedekt pensioensysteem aanwezig is. Daarom richten we ons op het bekendere evenwicht in de rest van de analyse.

In de numerieke analyse laten we zien dat, afhankelijk van de oorzaak van demografische veranderingen, de per-capita kapitaalgoederenvoorraad kan toenemen, afnemen of niet veranderen als gevolg van een toename van de bevolkingsgroei. Als de bevolkingsgroei toeneemt wegens een toename in het geboortecijfer, moet de kapitaalgoederenvoorraad over meer mensen verdeeld worden waardoor de per-capita kapitaalgoederenvoorraad afneemt. Als echter de bevolkingsgroei toeneemt vanwege een afname in het sterftecijfer, zullen individuen meer sparen waardoor de per-capita kapitaalgoederenvoorraad toeneemt. Ten slotte, als de bevolkingsgroei verandert door een combinatie van het geboorte- en het sterftecijfer, kan de per-capita kapitaalgoederenvoorraad ook exact gelijk blijven.

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Annuities, Public Policy and Demographic Change in Overlapping Generations Models

Using overlapping generations models this thesis analyses annuity markets, public policy and demographic change. The main findings are that annuities may not be welfare maximising after all, that raising the retirement age could actually decrease economic growth and that demographic changes influence the economy in a variety of opposing ways depending on the source of demographic change.

Throughout the thesis, we have aimed to highlight the usefulness of small models for macroeconomic analysis. In these models, a conscious trade-off is being made between sound micro-economic foundations and the ability to see what mechanisms are at play in the interaction between individual decisions and macroeconomic outcomes. We have been able to show that theoretical as well as policy issues can be studied meaningfully with such small models.

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